Exercise 1: Regular Expressions (6 Points)

Consider the following regular expressions. What language do they recognize? Give two strings that are members of the corresponding language and two strings which are not members – a total of four strings for each part. Assume for the first two parts that the alphabet $\Sigma = \{a, b, c\}$.

(a) $a^*b^*c^*$

(b) $((a \cup c)^*b(a \cup c)^*b(a \cup c)^*b(a \cup c)^*)^*$

Give a regular expression for each of the following languages.

(c) $L_1$ is the language, over alphabet $\{a, b\}$, of all strings starting and ending with the same symbol.

(d) $L_2$ is the language, over alphabet $\{0, 1\}$, of all alternating 0 and 1 strings.

Exercise 2: The Pumping Lemma: Sufficiency or Necessity? (4 Points)

Consider the language $L = \{c^m a^n b^n | m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

(a) Describe in words (not using the pumping lemma), why $L$ cannot be a regular language.

(b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use $L$ as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: To Be Regular or Not to Be (6 Points)

Let $\Sigma = \{0, 1\}$, prove the following:

(a) The language $A = \{0^k w 0^k | k \geq 1 \text{ and } w \in \Sigma^*\}$ is regular.

(b) The language $B = \{0^k 1 w 0^k | k \geq 1 \text{ and } w \in \Sigma^*\}$ is not regular.
Exercise 4: NFAs to Regular Expressions (4 Points)

Consider the following NFA:

Give the regular expression defining the language recognized by this NFA by stepwise converting it into an equivalent GNFA with only two nodes.