

Theoretical Computer Science - Bridging Course

Summer Term 2020

Exercise Sheet 4

for getting feedback submit electronically by 12:15 p.m., Tuesday, June 9th, 2020

Exercise 1: Context-Free Grammar (2+3 Points)

For each of the following languages, give a context-free grammar to accept the language. The alphabet set is $\Sigma = \{0, 1\}$.

- (a) $\{w \mid w \text{ contains at least three ones}\}$.
- (b) $L_1 = \{w\#w' \mid w^R \text{ is a substring of } w', \text{ and } w, w' \in \{0, 1\}^*\}$.¹

Exercise 2: Chomsky Normal Form (1+4 Points)

Consider the following context-free grammar (CFG):

$$\begin{aligned} S &\rightarrow aSb \mid D \\ D &\rightarrow ccDc \mid \varepsilon \end{aligned}$$

- a) Which language does this grammar define?
- b) Convert this CFG into an equivalent one in Chomsky Normal Form. Give the grammar you obtained after each step of the conversion algorithm.

Exercise 3: Constructing Pushdown Automata (4 Points)

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$. Construct a PDA \mathcal{A} with $L(\mathcal{A}) = L$.

Exercise 4: Pumping Lemma for Context-Free Languages (3+3 Points)

Use the pumping lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not context free:

- (a) $\{ww \mid w \in \{a, b\}^*\}$
- (b) $\{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$

¹ w^R is achieved by reversing the order of the symbols in w .