Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2020
Exercise Sheet 5

for getting feedback submit electronically by 12:15 pm, Tuesday, June 16, 2020

Exercise 1: The Shift Operation *(4+4 Points)*

Consider a Turing machine $\mathcal{M}$ that is given an arbitrary input string over alphabet $\Sigma = \{1, 2, \ldots, n\}$ on its input tape. We would like $\mathcal{M}$ to insert an empty cell, i.e., $\sqcup$, at the beginning of the tape without removing any symbol on the tape. As an example, the Turing machine is supposed to change the input tape of the form $\langle 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$ to $\langle \sqcup, 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$. Although this operation is not explicitly defined for a Turing machine, one can consider such an operation as shifting the whole string one cell to the right on the input tape.

(a) Give a formal definition of $\mathcal{M}$ to perform the desired operation such that $\mathcal{M}$ recognizes the language Σ^*.

(b) For $n = 2$, i.e., $\Sigma = \{1, 2\}$, draw the state diagram of your constructed Turing machine.

Exercise 2: Constructing Turing Machines I *(4+1+2+1 Points)*

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \ldots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \ldots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over Σ.

(a) Give a state diagram of a Turing machine recognizing L.

(b) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Constructing Turing Machines II *(4 Points)*

Let $L = \{\langle w \rangle, \langle w + 1 \rangle \mid w \in \mathbb{N}\}$, e.g., the word $\langle 6 \rangle, \langle 7 \rangle = 110, 111$ is contained in L. Design a Turing machine which accepts L. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.
Remark: Here $\langle w \rangle$ is the binary encoding of the number w, e.g., the number 6 is going to be the string 110.