Exercise 1: Constructing Turing Machines  

(a) \( L_1 = \{a^i b^i a^j b^j | i, j > 0 \} \)  
(b) Language \( L_2 \) of all strings over alphabet \( \{a, b\} \) with the same number of \( a \)'s and \( b \)'s.

Remark: It is sufficient to give a detailed description of the Turing Machines. You do not need to give formal definitions.

Exercise 2: Semi-Decidable vs. Recursively Enumerable  

(a) Show that any recursively enumerable language is semi-decidable.  
(b) Show that any semi-decidable language is recursively enumerable.

Exercise 3: Halting Problem  

The special halting problem is defined as  
\[ H_s = \{ \langle M \rangle | \langle M \rangle \text{ encodes a TM and } M \text{ halts on } \langle M \rangle \} \].

(a) Show that \( H_s \) is undecidable.  

Hint: Assume that \( M \) is a TM which decides \( H_s \) and then construct a TM which halts iff \( M \) does not halt. Use this construction to find a contradiction.

(b) Show that the special halting problem is recursively enumerable.

(c) Show that the complement of the special halting problem is not recursively enumerable.  

Hint: What can you say about a language \( L \) if \( L \) and its complement are recursively enumerable? (if you make some observation for this, also prove it)

(d) Let \( L_1 \) and \( L_2 \) be recursively enumerable languages. Is \( L_1 \setminus L_2 \) recursively enumerable as well?

(e) Is \( L = \{ w \in H_s | |w| \leq 1742 \} \) decidable? Explain your answer!