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# Theoretical Computer Science - Bridging Course
# Summer Term 2020
# Exercise Sheet 7

**for getting feedback submit electronically by 12:15 pm, Tuesday, June 30th, 2020**

## Exercise 1: Decidability *(7 Points)*

Let $\Sigma$ be a fixed finite alphabet. Show that the language of deterministic finite automatons (DFAs) on $\Sigma$ that accept every word is decidable. Formally, show that

$$L = \{\langle A \rangle \mid A \text{ is a deterministic finite automaton with } L(A) = \Sigma^*\}$$

is a decidable language.

*Remark: You can use that it is not difficult to construct a Turing machine which tests whether an input is the well formed encoding of a deterministic finite automaton.*

## Exersive 2: Landau Notation *(2+2+3 Points)*

The set $\mathcal{O}(f)$ contains all functions that are asymptotically not growing faster than the function $f$ (when additive or multiplicative constants are neglected). That is:

$$g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)$$

For the following pairs of functions, check whether $f \in \mathcal{O}(g)$ or $g \in \mathcal{O}(f)$ or both. Prove your claims (you do not have to prove a negative result $\notin$, though).

(a) $f(n) = 100n,\ g(n) = 0.1 \cdot n^2$

(b) $f(n) = \sqrt[3]{n^2},\ g(n) = \sqrt{n}$

(c) $f(n) = \log_2(2^n \cdot n^3),\ g(n) = 3n$ ***Hint**: You may use that $\log_2 n \leq n$ for all $n \in \mathbb{N}$.*

## Exercise 3: Sorting Functions by Asymptotic Growth *(6 Points)*

Sort the following functions by asymptotic growth using the $\mathcal{O}$-notation. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $f \notin \mathcal{O}(g)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

| | | | |
|---|---|---|---|
| $n^2$ | $\sqrt{n}$ | $2^n$ | $\log(n^2)$ |
| $3^n$ | $n^{100}$ | $\log(\sqrt{n})$ | $(\log n)^2$ |
| $\log n$ | $10^{100}n$ | $n!$ | $n \log n$ |
| $n \cdot 2^n$ | $n^n$ | $\sqrt{\log n}$ | $n$ |