Exercise 1: Binary Search Trees I

Consider the following binary search tree.

1. Give all sequences of insert(key) operations that generate the tree.

2. Draw the tree after the following sequence of operations: insert(6), insert(5), remove(3).

Exercise 2: Binary Search Trees II

(a) Describe a function that takes a binary search tree $B$ and a key $x$ as input and generates the following output:

- If there is an element $v$ in $B$ with $v.key = x$, return $v$.
- Otherwise, return the pair $(u, w)$ where $u$ is the tree element with the next smaller key and $w$ is the element with the next larger key. It should be $u = None$ if $x$ is smaller than any key in the tree and $w = None$ if $x$ is larger than any key in the tree.

For your description you can use pseudo code or a sufficiently detailed description in English. Analyze the runtime of your function.

(b) Describe a function which returns the depth of a binary search tree and analyze the runtime.

(c) Describe a function that for a given binary search tree with $n$ nodes and a given $k \leq n$ returns a list with the $k$ smallest keys from the tree. Analyze the runtime.