Exercise 1: Red-Black Trees

(a) Decide for each of the following trees if it is a red-black tree and if not, which property is violated:

(b) On the following red-black tree, first execute the operation \texttt{insert}(8) and afterwards \texttt{delete}(5).

Draw the resulting tree and document intermediate steps.

Exercise 2: AVL-Trees

An AVL-tree is a binary search tree with the additional property that for each node \( v \), the depth of its left and its right subtree differ by at most 1.

(a) Show via induction that an AVL-tree of height \( d \) is filled completely up to depth \( \lfloor \frac{d}{2} \rfloor \).

A binary tree is filled completely up to depth \( d' \) if it contains for all \( x \leq d' \) exactly \( 2^x \) nodes of depth \( x \).
(b) Give a recursion relation that describes the minimum number of nodes of an AVL-tree as a function of \( d \).

(c) Show that an AVL-tree with \( n \) nodes has depth \( \mathcal{O}(\log n) \).

*You can either use part (a) or part (b).*