The CONGEST model

The CONGEST model is a synchronous message passing model where the size of each message is only allowed to contain bitstrings of length $O(\log n)$, where $n$ is the number of nodes (do not confuse message size with message complexity). Further, we assume that the nodes have IDs in \{1, \ldots, n\}.

This means that each message may contain for example (the binary representation of) a constant number of integers $\leq n^c$ for some constant $c$ (in particular IDs). However, a node can not send the IDs of all its neighbors in a single message, as the degree of the network may be large.

Exercise 1: Leader Election

(10 Points)

a) Given a graph $G$, describe a deterministic algorithm in the CONGEST model such that every node learns the smallest ID in the graph and terminates after $O(D)$ rounds. You may not assume that nodes initially know $D$.

b) Analyze the message complexity of the algorithm. Show that your bound is tight.

Exercise 2: $k$-Selection Problem in Graphs

(10 Points)

Given a graph $G$ with $n$ nodes that have pairwise distinct input values $\leq n^c$ for some constant $c$. In order to solve the $k$-selection problem in the distributed setting for some $k \leq n$, the $k^{th}$-smallest value in the graph needs to be announced by exactly one node.

Our goal is to describe a randomized distributed algorithm in the CONGEST model that always solves the $k$-selection problem with an expected runtime of $O(D \cdot \log n)$.

a) Assume a tree $T$ of depth $D$. Describe an algorithm that computes in $O(D)$ rounds for every node $v$ a value $s_v$ which equals the size (number of nodes) of the subtree with root $v$.

b) Assume a tree $T$ of depth $D$ and root $v$ in which each node is able to flip coins. Describe a method to choose a node from the tree uniformly at random (i.e., each node has the same probability to be chosen) in time $O(D)$.

    Hint: Use the algorithm from a).

c) Assume a tree $T$ of depth $D$, where each node $v$ a boolean $b_v$ as input. Modify the algorithm of a) such that for every node $v$, the value $s_v$ is equal to the number of nodes in the subtree rooted at $v$ that have $b_v = \text{True}$. Also, modify the algorithm from b) to choose uniformly at random a node among all nodes $v$ with $b_v = \text{True}$.

d) Describe a randomized algorithm that solves the $k$-selection problem with an expected runtime of $O(D \cdot \log n)$.

    Hint: Use the previous algorithms.