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Dept. of Computer Science
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Theoretical Computer Science - Bridging Course
Exercise Sheet 1

Due: Tuesday, 27th of April 2021, 12:00 noon

Exercise 1: Proof by Induction *(5 Points)*

Prove the following summation formulas for all $n \in \mathbb{N}$ (i.e., all integers bigger 0) by induction on n.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Exercise 2: Relations between Sets *(5 Points)*

State whether the following set relations are true or false. Prove your claim.

(a) $\overline{\mathcal{A} \cup \mathcal{B}} \stackrel{?}{=} \overline{\mathcal{A}} \cap \overline{\mathcal{B}}$

(b) $\overline{\mathcal{A} \cup \mathcal{B}} \stackrel{?}{\subseteq} \overline{\mathcal{A} \cap \mathcal{B}}$

(c) $\{n^3 + 2n \mid n \in \mathbb{N}\} \stackrel{?}{=} \{n \mid n \in \mathbb{N} \text{ is divisible by } 3\}$

(d) $\{n^3 + 2n \mid n \in \mathbb{N}\} \stackrel{?}{\subseteq} \{n \mid n \in \mathbb{N} \text{ is divisible by } 3\}$

Remark: Assume $\mathcal{A}, \mathcal{B} \subseteq \Omega$ are two subsets from a groundset Ω (the set of "all" elements). To disprove a relation from above it is sufficient to give concrete examples for $\mathcal{A}, \mathcal{B}, \Omega$ for which the relation does not hold (i.e., a counterexample).

Exercise 3: Counting Edges in Acyclic Graphs *(5 Points)*

A tree is an acyclic, connected, simple graph. Show by induction that a tree with $n \geq 1$ nodes has $n-1$ edges. A forest is an acyclic, simple graph (not necessarily connected, i.e., it may consist of several unconnected trees). Show that a forest consisting of k connected components has $n-k$ edges.

Remark: Acyclic means that there are no cycles. A simple graph has no self-loops or multi-edges.

Exercise 4: Nodes with Identical Degrees *(5 Points)*

Show that every simple graph with two or more nodes contains two nodes with the same degree.

Hint: A proof by contradiction is useful.