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Theoretical Computer Science - Bridging Course
Exercise Sheet 3

Due: Tuesday, 12th of May 2021, 12:00 noon

Exercise 1: Regular Expressions *(5 Points)*

Regular expressions define *languages*, i.e., *sets* of words. For the following pairs of *languages* over the alphabet $\Sigma = \{a, b, c\}$, state whether one contains the other, or both, or neither. Prove your claim. For languages given in *set-notation* give a regular expression that defines *the same* language.

a) $(abc)^*$, $(a \cup b \cup c)^*$

b) $\Sigma^+ \cup ab$, $\Sigma^* ab \Sigma^* \cup a$

c) $\{x \in (abc)^* \mid |x| = 4y, y \in \mathbb{N}\}$, $(\Sigma\Sigma\Sigma\Sigma)^*$ *Remark: $|x|$ is the length of $x \in \Sigma$*

d) $\{xL'x \mid x \in \Sigma, L \in \Sigma^*\}$, $abc\Sigma^*cba$

Exercise 2: Limits of the Pumping Lemma *(5 Points)*

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

a) Describe in words (not using the pumping lemma), why L can not be a regular language.

b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Applications of the Pumping Lemma *(5 Points)*

Use the Pumping Lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not regular.

a) $L = \{a^m \mid m = n^2 \text{ for some } n \in \mathbb{N}\}$

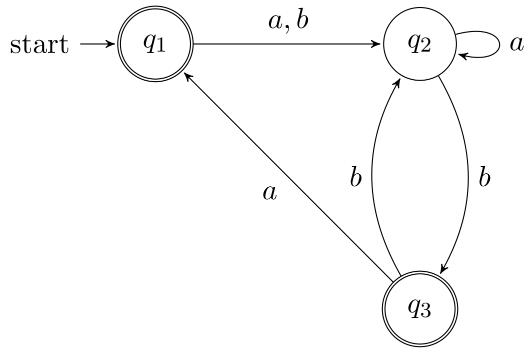
b) $L = \{a^k \mid k \text{ is prime}\}$

c) $L = \{a^m b^n \mid m \neq n\}$

Hint: Have a look at the languages $\{a^n b^n \mid n \in \mathbb{N}\}$ and a^*b^* and use the fact that regular languages are closed under regular operations.

Exercise 4: GNFA *(5 Points)*

Consider the following NFA:



Give the regular expression defining the language recognized by this NFA by converting it *stepwise* into an equivalent GNFA with only two nodes. Document your steps.