Exercise 1: Regular Expressions

Regular expressions define languages, i.e., sets of words. For the following pairs of languages over the alphabet \( \Sigma = \{a, b, c\} \), state whether one contains the other, or both, or neither. Prove your claim. For languages given in set-notation give a regular expression that defines the same language.

a) \((abc)^*, (a \cup b \cup c)^*\)

b) \(\Sigma^+ \cup ab, \Sigma^*ab\Sigma^* \cup a\)

c) \(\{x \in (abc)^* \mid |x| = 4y, y \in \mathbb{N}\}, (\Sigma\Sigma\Sigma\Sigma)^*\)

Remark: \(|x|\) is the length of \(x \in \Sigma\)

d) \(\{xL'x \mid x \in \Sigma, L \in \Sigma^*\}, abc\Sigma^*cba\)

Exercise 2: Limits of the Pumping Lemma

Consider the language \(L = \{c^ma^n \mid m, n \geq 0\} \cup \{a, b\}^*\) over the alphabet \(\Sigma = \{a, b, c\}\).

a) Describe in words (not using the pumping lemma), why \(L\) can not be a regular language.

b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use \(L\) as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Applications of the Pumping Lemma

Use the Pumping Lemma to show that the following languages over the alphabet \(\Sigma = \{a, b\}\) are not regular.

a) \(L = \{a^m \mid m = n^2 \text{ for some } n \in \mathbb{N}\}\)

b) \(L = \{a^k \mid k \text{ is prime}\}\)

c) \(L = \{a^mb^n \mid m \neq n\}\)

Hint: Have a look at the languages \(\{a^n b^n \mid n \in \mathbb{N}\}\) and \(a^*b^*\) and use the fact that regular languages are closed under regular operations.
Exercise 4: GNFA (5 Points)

Consider the following NFA:

Give the regular expression defining the language recognized by this NFA by converting it stepwise into an equivalent GNFA with only two nodes. Document your steps.