



Theoretical Computer Science - Bridging Course

Exercise Sheet 4

Due: Wednesday, 19th of May 2021, 12:00 pm

Exercise 1: Context Free Grammar

(7 Points)

Give a context free grammar for each of the following languages, where the alphabet set for the first three is $\Sigma = \{a, b\}$.

1. $L_1 = \{a^k b^{3k} \mid k \geq 0\}$
2. $L_2 = \{a^i b^j \mid 0 < i \leq j\}$
3. $L_1 \cup L_2$
4. $L_3 = \{w \in \{0, 1\}^* \mid w \text{ contains at least three ones}\}$

Exercise 2: Chomsky Normal Form

(4 Points)

Convert the following CFG into an equivalent CFG in Chomsky Normal Form (CNF). Write down the grammar you obtain after each step of the conversion algorithm.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \epsilon \\ B &\rightarrow 00 \mid \epsilon \end{aligned}$$

Exercise 3: Constructing Pushdown Automata

(3 Points)

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$. Construct a PDA \mathcal{A} with $L(\mathcal{A}) = L$.

Exercise 4: Pumping Lemma for Context-Free Languages (3+3 Points)

Use the pumping lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not context free:

- (a) $L_1 = \{a^m \mid m \text{ is a prime}\}$
- (b) $L_2 = \{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$