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Dept. of Computer Science
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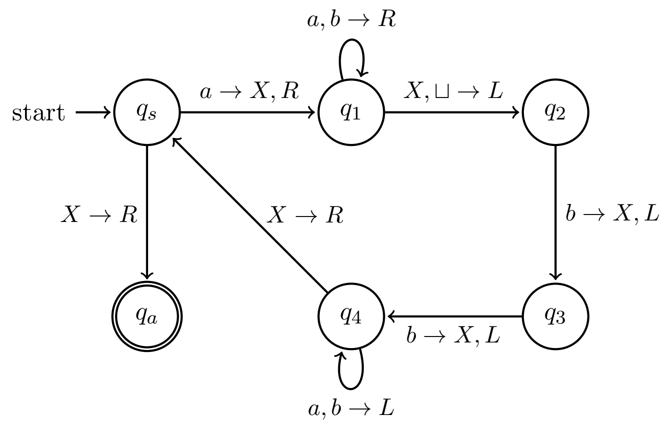
Theoretical Computer Science - Bridging Course
Exercise Sheet 5

Due: Wednesday, 2nd of June 2021, 12:00 noon

Exercise 1: Turing Machine as State Diagram *(7 Points)*

Consider the Turing machine M over the alphabet $\Sigma = \{a, b\}$, which is given via the following state diagram. *Remark: The blank symbol* $\sqcup \in \Gamma$ *represents an empty cell on the tape.*



a) Simulate M with input $s_1 = aabbbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_1 \in L$.

b) Simulate M with input $s_2 = aabbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_2 \in L$.

c) Give a description of the language $L(M)$ that M recognizes in the form of a set.

Exercise 2: Turing Machine to Recognize a Language *(8 Points)*

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes.

a) Give a state diagram of a Turing machine that recognizes L.

b) Give the maximum number of head movements (or a close upper bound) your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

d) Give the maximum number of head movements (or a close upper bound) your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Turing Machine to Compute a Function *(5 Points)*

A Turing machine computes a function $f : \Sigma^* \to \Sigma^*$, if for any input $w \in \Sigma^*$ it creates the string $f(w)$ on its tape (and nothing else can be on the tape) and halts. Give an informal "mid-level"-description (c.f. lecture) of a Turing machine over $\Sigma = \{0, 1\}$ that gets a bit string of a number $n \in \mathbb{N}$ as input and computes the function $f(n) = 2(n + 1)$ (output again as bit string).