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Theory of Distributed Systems
Exercise Sheet 1

Due: Wednesday, 24th of April 2024, 12:00 noon

Exercise 1: Schedules and Correctness Properties *(5 Points)*

(a) Consider three nodes, v_1, v_2, and v_3, which are connected via FIFO channels, that is, messages between any two nodes are received in the same order they are sent. For example, if node v_1 sends first message m_1 then m_2 to node v_2, then v_2 will first receive m_1 and then m_2.

Devise **one** possible schedule S which is consistent with the following local restrictions to the three nodes.

- $S|1 = s_{1,3}\ s_{1,3}\ r_{1,2}\ r_{1,3}\ s_{1,2}\ r_{1,2}\ s_{1,3}$,
- $S|2 = s_{2,3}\ s_{2,1}\ r_{2,1}\ s_{2,1}$,
- $S|3 = r_{3,2}\ r_{3,1}\ s_{3,1}\ r_{3,1}\ r_{3,1}$.

$s_{i,j}$ denotes the send event from node i to node j and $r_{j,i}$ denotes the event that node j receives a message from node i.

(b) Consider that the communication service above ensures that messages exchanged between the nodes are:

- never lost
- never duplicated
- received in the same order they are sent

Specify for each which correctness property it represents.

Exercise 2: The Level Algorithm *(5 Points)*

Consider the following algorithm between two connected nodes u and v:

The two nodes maintain levels ℓ_u and ℓ_v, which are both initialized to 0. One round of the algorithm works as follows:

1. Both nodes send their current level to each other
2. If u receives level ℓ_v from v, u updates its level to $\ell_u := \max\{\ell_u, \ell_v + 1\}$. If the message to node u is lost, node u does not change its level ℓ_u. Node v updates its level ℓ_v in the same (symmetric) way.

Argue that if the level algorithm runs for r rounds, the following properties hold:

(a) At the end, the two levels differ by at most one.

(b) If all messages succeed, both levels are equal to r.

(c) The level of a node is at least 1 if and only if the node received at least one message.

Exercise 3: (Variations) of Two Generals *(10 Points)*

In the lecture we considered the (deterministically unsolvable) **Two Generals** consensus problem:

- two deterministic nodes, synchronuous communication, unreliable messages,
- **input**: 0 or 1 for each node,
- **output**: each node needs to decide either 0 or 1,
- **agreement**: both nodes must output the same decision (0 or 1),
- **validity**: if both nodes have the same input $x \in \{0, 1\}$ and no messages are lost, both nodes output x,
- **termination**: both nodes terminate in a bounded number of rounds.

(a) Generalize the problem to an arbitrary number $n \geq 2$ of generals i.e. whenever there are two nodes in the problem description above, we replace it with n nodes. Show that even with this generalization the problem remains deterministically unsolvable.

For the rest of the exercise, we consider three modifications of the model. For each of them, either give a (deterministic) algorithm or state a proof which shows that the variation cannot be solved deterministically.

(b) There is the guarantee that within the first 10 rounds at least *one* message in *each* direction succeeds.

(c) There is the guarantee that within the first 10 rounds at least *one* message succeeds.
***Remark**: nodes are not allowed to stay silent.*

(d) Let $k \in \mathbb{N}$ be a natural number. The input for each node is a number $x \in \{0, \ldots, k\}$.

Goal: If no message gets lost *and* both have the same input $x \in \{0, \ldots, k\}$, both have to output x. In all other cases the nodes should output numbers which do not differ by more than one. The algorithm still has to terminate in a finite number of rounds.