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Theoretical Computer Science - Bridging Course
Exercise Sheet 3

Due: Tuesday, 7th of May 2024, 12:00 pm

Exercise 1: REs *(2+2+2+2 Points)*

(a) Let $\Sigma = \{a, b\}$. Let L_1 be the language defined by the regular expression $a^*b^*a^*$ and L_2 the language defined by a^*b^*b. Draw a DFA for L_1 and L_2.

(b) Let $\Sigma = \{a, b, c\}$. What language does the following regular expression describe $((a \cup c)^*b(a \cup c)^*b(a \cup c)^*b(a \cup c)^*)^*$?

(c) Let $\Sigma = \{a, b\}$. Provide a regular expression that recognizes the following two languages.

- Let language L_3 contain all strings in which at least one of the symbols a or b occurs an even number of times.
- Let language L_4 contain all strings of length at least 2 such that a and b are alternating.

Exercise 2: Limits of the Pumping Lemma *(1+3 Points)*

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet set $\Sigma = \{a, b, c\}$.

(a) Describe in words (not using the pumping lemma), why L can not be a regular language.

(b) Show that, while the property described in the Pumping Lemma is a necessary condition for regularity, it is *not* sufficient for regularity.

Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Proving Non-regularity *(2+3 Points)*

Use the Pumping Lemma to show that the following languages over the alphabet set $\Sigma = \{a, b, c\}$ are not regular.

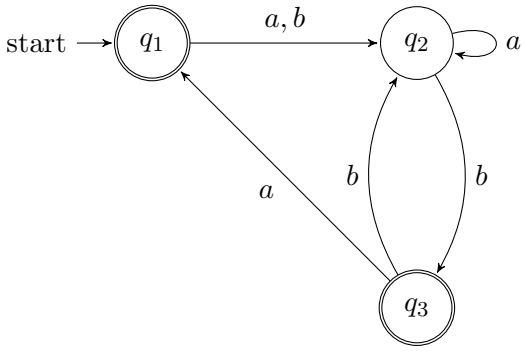
(a) $L := \{a^n c b^{n+2} \mid n \geq 0\}$.

(b) $L = \{a^m \mid m = n^2 \text{ for some } n \geq 0\}$.

Bonus: $L = \{a^n bwa^n | n \geq 1 \text{ and } w \in \Sigma^*\}$.

Exercise 4: NFA-GNFA-RE *(3 Points)*

Consider the following NFA:



Give the regular expression defining the language recognized by this NFA by converting it *stepwise* into an equivalent GNFA with only two nodes. Document your steps.