



Theoretical Computer Science - Bridging Course

Exercise Sheet 4

Due: Tuesday, 14th of May 2024, 12:00 pm

Exercise 1: CFGs and PDAs

(5+3 Points)

Give a context free grammar for each of the following languages.

- $L_1 = \{a^i b^j \mid 0 < i \leq j\}$
 - $L_2 = \{a^{2n} b^n \mid n > 0\}$
 - $L_3 = \{a^* w c^k \mid w \in \{a, b\}^*, \text{ and } k \text{ is the number of } a\text{'s in } w\}$
- Create a pushdown automaton that accepts languages L_2 and L_3 .

Exercise 2: Proving NonCFL

(4+4 Points)

Use the Pumping Lemma to show that the following languages are not CFL.

- $L_4 = \{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$
- $L_5 = \{a^i b^j c^k \mid i < j \text{ and } i < k\}$

Bonus: $L_6 = \{a^m \mid m \text{ is a prime}\}$

NB: If you wish you can try first and prove it nonregular using the Pumping Lemma for regular languages and the same idea should be extended to CFLs.

Exercise 3: Closure in CFL

(2+2 Points)

- Show that the context-free languages are closed under union, concatenation and Kleene star.
Hint: try to prove that the context-free languages are closed under the above operators via creating the appropriate grammars.
- Knowing that $L_7 = \{a^i b^j c^k \mid i < j\}$ is a context free language, are context free languages closed under intersection?
Hint: Use the fact that L_5 is not a context free language.