Albert-Ludwigs-Universität
Institut für Informatik
Prof. Dr. F. Kuhn

Exam Algorithm Theory

Thursday, August 30 2012, 10:00 – 11:30

Name: ....

Matriculation Nr.: ....

Signature.: ....

Do not open or turn until told so by the supervisor!

Instructions:

- Write your name and matriculation number on the cover page of the exam and sign the document! Write your name on all sheets!
- Your signature confirms that you have answered all exam questions without any help, and that you have notified exam supervision of any interference.
- Write legibly and only use a pen (ink or ball point). Do **not** use **red**! Do **not** use a pencil!
- You are **not** allowed to use any material except for a pocket calculator and a self-written summary of at most 2 pages.
- There are 4 questions and there is a total of 90 points. The number of points for each question is given.
- Only one solution per question is graded! Make sure to strike out any solutions that you do not want to be considered!

- **Do not leave your seat at the end of the examination until all exams have been collected!**

Question	Achieved Points	Max Points
1		34
2		18
3		23
4		15
Total		90

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Problem 1: Short Questions **34 points**

Answer the following questions. Explain your answers!

(a) **(5 points)** Is $(\log n)!$ in $\Omega(n^2)$?

(b) **(5 points)** Describe how to use a priority queue for sorting n numbers! Why does this imply that for each (comparison-based) priority queue implementation, at least one of the two operations “insert” and “delete-min” has cost $\Omega(\log n)$?

(c) **(3 points)** What does it mean that some data structure operation has amortized cost $a(n)$?

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(d) **(4 points)** Does Prim's MST algorithm work if the input graph can have positive and negative edge weights?

(e) **(4 points)** What is the discrete Fourier transform of a vector $(a_0, a_2, \ldots, a_{n-1})$? (definition suffices)

(f) **(6 points)** What is the asymptotic time complexity of an algorithm with recursion formula $T(1) = 1, T(n) = 4T(n/2) + n$?

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(g) **(7 points)** By what factor can the total weight of a shortest path tree and a minimum spanning tree differ? Try to give asymptotically tight bounds!
Thus, show that for some $\alpha(n)$ and every shortest path tree T, $weight(T) \in O(\alpha(n) \cdot weight(MST))$, and give a shortest path tree T for which $weight(T) \in \Omega(\alpha(n) \cdot weight(MST))$.

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Problem 2: Priority Queues **18 points**

a) **(10 points)** Describe and compare the properties of binomial heaps and Fibonacci heaps! What can you say about the structure of binomial heaps and Fibonacci heaps? What do they have in common and in what way do they differ? Also compare binomial heaps and Fibonacci heaps in terms of running times! Can you think of scenarios in which it is better to use a binomial heap and scenarios in which it is better to use a Fibonacci heap?

b) **(8 points)** Perform the following sequence of operations on an initially empty binomial heap! Draw the state of the binomial heap after each of the operations!

1. insert(10)
2. insert(3)
3. insert(5)
4. insert(2)
5. insert(7)
6. decrease-key(10,1)
7. insert(4)
8. delete-min()
9. delete-min()
10. delete-min()

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Problem 3: Dynamic Programming **23 points**

(a) **(6 points)** Briefly describe what dynamic programming is and how it works (try to use not more than 3-4 sentences)! What are the types of problems to which dynamic programming can be applied?

(b) **(6 points)** Consider a sequence $A = a_1, a_2, \ldots, a_n$ of characters. A sub-sequence S of A is a sequence that can be obtained by deleting some characters in A. Hence, for example, A, R, E, W, A, A is a sub-sequence of $B, A, A, R, E, Z, W, A, B, A$. Given two sequences A of length n and B of length m, describe an $O(m \cdot n)$ time algorithm to compute the length of a longest common sub-sequence of A and B!

(c) **(8 points)** Use your algorithm to compute the length of a longest common sub-sequence of the sequences A, E, A, A, G, S, S, T and B, A, E, B, S, A, G, T (give all the intermediate results of the algorithm)!

(d) **(3 points)** How do you have to change your algorithm to also return a longest common sub-sequence (and not just its length)?

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Problem 4: Distribution of Sum **15 points**

Let A and B be two sets of integers between 0 and n, i.e., $A, B \subseteq \{0, \dots, n\}$. We define two random variables X and Y, where X is obtained by choosing a number uniformly at random from A and Y is obtained by choosing a number uniformly at random from B. We further define the random variable $Z = X + Y$. Note that the random variable Z can take values from the range $\{0, \dots, 2n\}$.

(a) **(6 points)** Give a simple $O(n^2)$ algorithm to compute the distribution of Z. Hence, the algorithm should compute the probability $\Pr(Z = z)$ for all $z \in \{0, \dots, 2n\}$.

(b) **(9 points)** Can you get a more efficient algorithm to compute the distribution of Z? You can use algorithms discussed in the lecture as a black box. What is the time complexity of your algorithm?

Hint: Try to represent A and B using polynomials.

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