

## Algorithm Theory, Winter Term 2014/15

### Problem Set 3

hand in (hard copied) by Thursday, 10:00, November 13, 2014, either before the lecture or in the box corresponding to your group in building no. 51.

#### Exercise 1: A Greedy TSP Approximation (6 points)

Consider a symmetrical ( $d(u, v) = d(v, u)$  for all  $u, v \in V$ ) TSP instance where all weights are either  $a$  or  $b$ , ( $0 < a < b$ ).

Prove that the nearest neighbor greedy algorithm from the lecture produces a  $(\frac{a+b}{2a})$ -approximation of an optimal TSP-tour.

#### Exercise 2: Matroids (3+1 points)

a) For a graph  $G = (V, E)$ , a subset  $F \subseteq E$  of the edges is called a forest iff (if and only if) it does not contain a cycle. Let  $\mathcal{F}$  be the set of all forests of  $G$ . Show that  $(E, \mathcal{F})$  is a matroid.

*Hint: A forest with  $k$  edges and  $n$  nodes has  $n - k$  connected components.*

b) For a matroid  $(E, I)$ , a maximal independent set  $S \in I$  is an independent set that cannot be extended. Thus, for every element  $e \in E \setminus S$ , the set  $S \cup \{e\} \notin I$ .

What are the maximal independent sets of the matroid in a)?