Albert-Ludwigs-Universität, Inst. für Informatik
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H. Ghodselahi, Y. Maus November 12, 2014

Algorithm Theory, Winter Term 2014/15
Problem Set 4

hand in (hard copied) by Thursday, 10:00, November 20, 2014, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: Knapsack (4+2+1 points)

In the lecture, we have considered the Knapsack problem: There are n items with positive weights $w_1, \dots, w_n$ and values $v_1, \dots, v_n$ and a knapsack (a bag) of capacity W. A feasible solution to the problem is a subset of the items such that their total weight does not exceed W. The objective is to find a feasible solution of maximum possible total value.
For the case where all weights are positive integers, we have discussed a dynamic programming solution that solves the knapsack problem in time $O(nW)$.

a) Assume that instead of the weights, the *values of all items are positive integers*. The weights of the items can be arbitrary positive real numbers. Describe a dynamic programming algorithm that solves the knapsack problem if all values are positive integers.

b) What is the running time of your algorithm? Justify your answer.

c) Knapsack is one of *Karp's* NP-complete problems. Both dynamic programming solutions lead to polynomial time algorithms. Why is this not a contradiction to the NP-completeness of Knapsack?

Exercise 2: Greedy for Knapsack (3 points)

You are given an instance of Knapsack, i.e., values $v_1, \dots, v_n$, weights $w_1, \dots, w_n$ and a capacity W.
Consider the following greedy algorithm to solve the problem:
Assume that the elements are given in decreasing order by value per weight unit, i.e.

$$v_1/w_1 \geq v_2/w_2 \geq \cdots \geq v_n/w_n.$$

The greedy algorithm starts with an empty knapsack. As long as there is space in the knapsack and there is an item that can be packed into the knapsack the greedy algorithm picks one of the items with the largest value per weight coefficient.
Show that this greedy algorithm can be arbitrarily bad.[1]

[1] Arbitrarily bad means that the ratio of the optimal solution to the greedy solution could be as large as possible.