Albert-Ludwigs-Universität, Inst. für Informatik
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December 3, 2014

Algorithm Theory, Winter Term 2014/15
Problem Set 7

hand in (hard copied) by Thursday, 10:00, December 11, 2014, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: MST Algorithm (4 points)

You are given a graph with average degree $\Theta\left(\sqrt{\log(n)}\right)$ **and all edge weights are integers between 1 and 10**. Does Kruskal's or Prim's algorithm have the better runtime? Explain your solution.

Exercise 2: Union-Find (3+3 points)

(a) In the lecture the union-by-size and the union-by-rank heuristics were introduced to implement a union-find data structure. The rank is *basically* the height of the tree. This is not true if we use path compression as the height of trees changes; but the rank is still an upper bound on the actual height of the tree. Give pseudo code for `union(x,y)`.

Show that when implementing a union-find data structure by using disjoint-set forests with the union-by-rank heuristic, the height of each tree is at most $O(\log n)$.

(b) Demonstrate that the above analysis is tight by giving an example execution (of merging n elements in that data structure) that creates a tree of height $\Theta(\log n)$. Can you even get a tree of height $\lfloor \log_2 n \rfloor$?