Albert-Ludwigs-Universität, Inst. für Informatik
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Algorithm Theory, Winter Term 2014/15
Problem Set 10

hand in (hard copied) by Thursday, 10:00, January 15, 2015, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: Matching (4.5+1.5 points)

Remark: This is a previous exam question.
A high school class is made an interesting offer by a reality TV show in which couples of students (one female and one male student) get a free vacation trip to an exciting location. This class consists of

- n boys $B = \{b_1, \ldots, b_n\}$ and
- n girls $G = \{g_1, \ldots, g_n\}$.

There are n different locations $L = \{\ell_1, \ldots, \ell_n\}$ for the students to choose from. The girls are not picky about the destinations, but each girl g is only willing to partner up with an individual subset $B_g \subseteq B$ of all available boys. The boys on the other hand do not care that much about with whom they go on vacation, but they care about the location; each boy b has an individual subset $L_b \subseteq L$ of locations it is willing to visit.

a) Is it possible that everyone can go on a free vacation? Devise an algorithm that answers this question.

b) What is the time complexity of your algorithm if you assume that each girl is willing to partner up with at most $\sqrt{n}$ different boys and if you assume that each boy is willing to to visit at most $\sqrt{n}$ different locations?

Exercise 2: Probability Theory (4 points)

A number of 20 customers go to the hairdresser's on some day. Assume that the arriving time of each client is exponentially distributed with mean 20 minutes. Further, suppose that the clients arrive independently of one another. Calculate the probability that for at least 15 customers the hairdresser has to wait at least half an hour till they arrive.
Hint: *Calculating the exact probability is a bit tedious; instead you are permitted to use Markov's inequality to provide a close upper bound.*