Albert-Ludwigs-Universität, Inst. für Informatik
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January 14, 2015

Algorithm Theory, Winter Term 2014/15
Problem Set 11

hand in (hard copied) by Thursday, 10:00, January 22, 2015, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: Randomized Primality Algorithms (4+1 points)

Remark: This is a question from a previous exam.
In the lecture, we have seen the Miller-Rabin randomized primality test. For a number N it tests whether N is a prime. If N is a prime, the test always returns "yes," if N is not prime, the test returns "no" with probability at least 3/4. The running time of the test is $\mathcal{O}(\log^2 N \cdot \log\log N \cdot \log\log\log N)$.

a) Your task now is to find an efficient randomized algorithm that for a given (sufficiently large) input number n, finds a prime number between n and $2n$. Your algorithm should succeed with probability at least $1 - 1/n$. You can assume that the number of primes between n and $2n$ is at least $\frac{n}{3\ln n}$.

b) Is your algorithm a Las Vegas or Monte Carlo one? What is the running time of your algorithm?

Exercise 2: Running Time of the Contraction Algorithm (5 points)

We have discussed the randomized contraction algorithm for the minimum cut problem in the lecture. When analyzing the algorithm, we have assumed that each contraction on a graph with n nodes can be done in time $O(n)$. Show that this is indeed possible.
Give an appropriate (simple) data structure to store the current multi-graph G and an algorithm that contracts a *given* edge in time $O(n)$.