Albert-Ludwigs-Universität, Inst. für Informatik
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Algorithm Theory, Winter Term 2014/15
Problem Set 13

hand in (hard copied) by Thursday, 10:00, February 5, 2015, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: Independent set on planar graphs (4* points)[1]

Design an approximation algorithm with constant approximation factor for the `INDEPENDENT SET` problem on planar graphs.
Hint: *You can use that in planar graphs the number of edges is limited such that* $|E| \leq 3|V| - 6$.

Exercise 2: Partition to obtain large Cuts (4 points)

You are given the following optimization problem:

- Input: A graph $G = (V, E)$,
- Solution: A partition of the set of nodes $V = V_1 \,\dot\cup\, V_2 \,\dot\cup\, V_3$,
- Goal: Maximize $|\{\{v_i, v_j\} \in E | v_i \in V_s, v_j \in V_t, s \neq t\}|$.

Design a 3/2-approximation algorithm. Prove that your algorithm is correct and gives the 3/2 approximation factor.

Please turn page for exercise 3

[1] With this exercise you can gain additional points to reach the 50% threshold to be admitted for the exam.

Exercise 3: Minimum Weight Vertex Cover (1+1+4 points)

Given is an undirect graph $G = (V, E)$, and a weight function on vertices $w : V \longrightarrow \mathbb{Q}^+$. The goal is to find a *minimum weight vertex cover*, i.e., a set $V' \subseteq V$, which minimizes $w(V') := \sum_{v \in V'} w(v)$ and ensures that every edge has at least one endpoint incident in V'. The special case, in which all vertices are unit weights, is called *cardinality vertex cover problem.*

a) First, provide a factor 2 approximation algorithm for the cardinality vertex cover problem using *maximal matching.*

Now we want to find a 2-approximation factor algorithm for the minimum weight vertex cover where vertices might have different weights.

Consider the following algorithm: In each iteration one arbitrary edge is picked up and the endpoints' weights are updated as follows

$$w_i(u) = w_{i-1}(u) - \min\{w_{i-1}(u), w_{i-1}(v)\} \text{ and}$$
$$w_i(v) = w_{i-1}(v) - \min\{w_{i-1}(u), w_{i-1}(v)\}.$$

Now, at least one of the endpoints has weight 0; we remove the node (both, if both have weight 0) and all its incident edges and move to the next iteration until there are no more edges to select.

b) With respect to the above algorithm, show that the removed vertices induce a vertex cover for G.

c) Prove that this algorithm gives us a factor 2 approximation for minimum weight vertex cover.

Hint: *Suppose that $S \subseteq V$ is the solution of the algorithm and $S^* \subseteq V$ is an optimal solution. Fill out the missing parts in*

$$w(S) = w(S \backslash (S \cap S^*)) + w(S \cap S^*) \leq \ldots \leq 2w(S^*). \quad (1)$$

Do not forget to argue your intermediate steps in detail.