Chapter 8
Online Algorithms

Algorithm Theory
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Online Computations

- Sometimes, an algorithm has to start processing the input before the complete input is known

- For example, when storing data in a data structure, the sequence of operations on the data structure is not known

Online Algorithm: An algorithm that has to produce the output step-by-step when new parts of the input become available.

Offline Algorithm: An algorithm that has access to the whole input before computing the output.

- Some problems are inherently online
  - Especially when real-time requests have to be processed over a significant period of time

Competitive Ratio

- Let's again consider optimization problems
  - For simplicity, assume, we have a minimization problem

Optimal offline solution $\mathbf{OPT}(I)$:

- Best objective value that an offline algorithm can achieve for a given input sequence I

Online solution $\mathbf{ALG}(I)$:

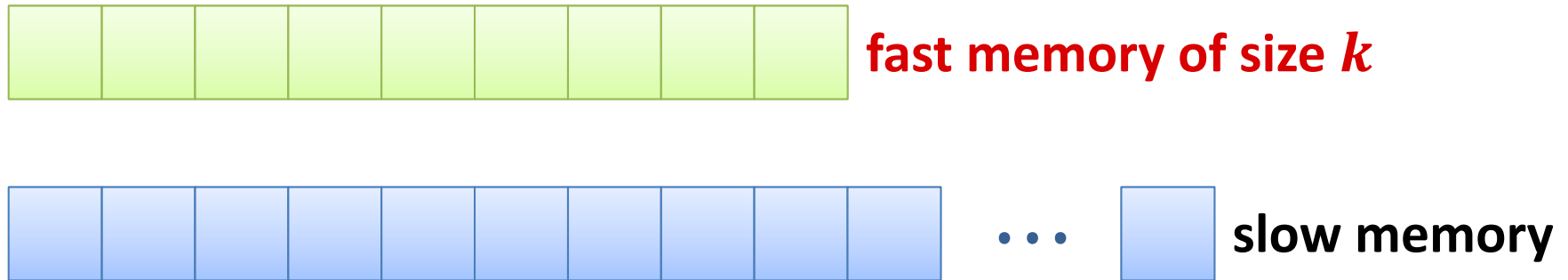- Objective value achieved by an online algorithm ALG on I

Competitive Ratio: An algorithm has competitive ratio $c \geq 1$ if

$$\mathbf{ALG}(I) \leq c \cdot \mathbf{OPT}(I) + \boldsymbol{\alpha}.$$

- If $\alpha \leq 0$, we say that ALG is strictly c-competitive.

Paging Algorithm

Assume a simple memory hierarchy:

fast memory of size k

$\cdots$ **slow memory**

If a memory page has to be accessed:

- Page in fast memory (hit): take page from there

- Page not fast memory (miss): leads to a page fault

- Page fault: the page is loaded into the fast memory and some page has to be evicted from the fast memory

- Paging algorithm: decides which page to evict

- Classical online problem: we don't know the future accesses

Paging Strategies

Least Recently Used (LRU):

- Replace the page that hasn't been used for the longest time

First In First Out (FIFO):

- Replace the page that has been in the fast memory longest

Last In First Out (LIFO):

- Replace the page most recently moved to fast memory

Least Frequently Used (LFU):

- Replace the page that has been used the least

Longest Forward Distance (LFD):

- Replace the page whose next request is latest (in the future)
- LFD is **not** an online strategy!