Albert-Ludwigs-Universität, Inst. für Informatik
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Algorithm Theory, Winter Term 2015/16
Problem Set 5

hand in (hard copy or electronically) by 10:15, Thursday November 26, 2015, tutorial session will be on November 30, 2015

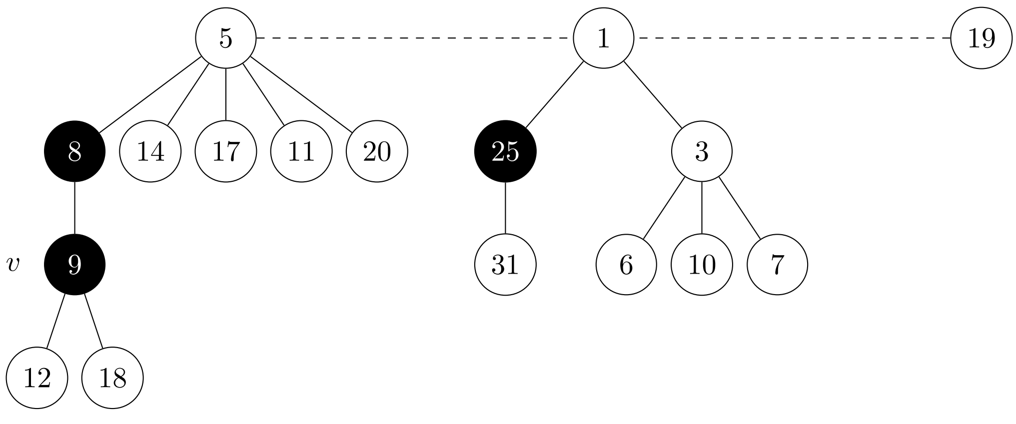
Exercise 1: Amortized Analysis (2+4 points)

(a) Consider an extension of the augmented stack data structure from the lecture: In addition to offering a *multipop*(k) operation, assume that the extended augmented stack also offers a *multipush*(k, L) operation which takes a parameter $k \geq 1$ and a list L of k elements and it pushes each of these elements on the stack. Assume that both operations *multipop*(k) and *multipush*(k, L) require time $\Theta(k)$ to complete. In the lecture, we proved that for the simple augmented stack, the amortized cost of all operations is $O(1)$. Does this $O(1)$ the amortized cost bound for all stack operations continue to hold for the extended version of the augmented stack? Explain your answer!

(b) Show how to implement a queue with two ordinary stacks so that the amortized cost of each *enqueue* and *dequeue* operations is $O(1)$. Explain your amortized analysis.

Exercise 2: Fibonacci Heaps (2+4 points)

(a) Consider the following Fibonacci heap (black nodes are marked, white nodes are unmarked). How does the given Fibonacci heap look after a *decrease-key*$(v, 2)$ operation and how does it look after a subsequent *delete-min* operation?



(b) Fibonacci heaps are only efficient in an amortized sense. The time to execute a single, individual operation can be large. Show that in the worst case, both the *delete-min* and the *decrease-key* operations can require time $\Omega(n)$ (for any heap size n).

Hint: *Describe an execution in which there is a delete-min operation that requires linear time and describe an execution in which there is a decrease-key operation that requires linear time.*