Albert-Ludwigs-Universität, Inst. für Informatik
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Algorithm Theory, Winter Term 2015/16
Problem Set 12

hand in (hard copy or electronically) by 10:15, Thursday January 28, 2016, tutorial session will be on February 1, 2016

Exercise 1: Covering as many Elements as Possible (8 points)

We consider the following variant of the set cover problem discussed in the lecture. We are given a set of elements X and a collection $\mathcal{S} \subseteq 2^X$ of subsets of X such that $\bigcup_{S \in \mathcal{S}} S = X$. In addition, we are given an integer parameter $k \geq 2$.

Instead of finding a collection $\mathcal{C} \subseteq \mathcal{S}$ of the sets which covers all elements, the goal is to find a set of (at most) k set $S_1, \dots, S_k \in \mathcal{S}$ such that the number of covered elements $|S_1 \cup \dots \cup S_k|$ is maximized.

We consider the greedy set cover algorithm from the lecture, but we stop the algorithm after adding k sets.

(a) (2 points) Show that for $k = 2$, the described greedy algorithm has approximation ratio at most 4/3.

(b) (4 points) Let us now consider a general parameter $k \geq 2$. Show that if an optimal choice of k sets $S_1, \dots, S_k$ covers ℓ elements, after adding t sets, the greedy algorithm covers at least $\frac{\ell}{k} \cdot \sum_{i=1}^{t} \left(1 - \frac{1}{k}\right)^{i-1}$ elements.

(c) (2 points) Prove that the approximation ratio of the greedy algorithm is at most $\frac{e}{e-1}$. You can use that $(1 - 1/k)^k < e^{-1}$.

Exercise 2: TSP in Graphs with Edge Weights 1 and 2 (4 points)

Consider the family of complete undirected graph G in which all edges have length either 1 or 2. Give a 4/3-approximation for the TSP problem for this family of graphs. Note that G satisfies the triangle inequality.

Hint: Start with a minimum 2-matching in G. A 2-matching is a subset M of edges so that every vertex in G is incident to exactly two edges in M. You can assume that a minimum 2-matching can be computed in polynomial time.