



Chapter 2

Greedy Algorithms

Algorithm Theory
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Fabian Kuhn

Matroids

- Same, but more abstract...

Matroid: pair (E, I)

- E : set, called the **ground set**
- I : finite family of finite subsets of E (i.e., $I \subseteq 2^E$), called **independent sets**

(E, I) needs to satisfy 3 properties:

1. Empty set is independent, i.e., $\emptyset \in I$ (implies that $I \neq \emptyset$)
2. **Hereditary property**: For all $A \subseteq E$ and all $A' \subseteq A$,
if $A \in I$, then also $A' \in I$
3. **Augmentation / Independent set exchange property**:
If $A, B \in I$ and $|A| > |B|$, there exists $x \in A \setminus B$ such that

$$\mathbf{B' := B \cup \{x\} \in I}$$

Matroids and Greedy Algorithms

Weighted matroid: each $e \in E$ has a weight $w(e) > 0$

Goal: find **maximum weight independent set**

Greedy algorithm:

1. Start with $S = \emptyset$
2. Add max. weight $e \in E \setminus S$ to S such that $S \cup \{e\} \in I$

Claim: **greedy algorithm** computes **optimal** solution

Matroids: Examples

Forests of a graph $G = (V, E)$:

- forest F : subgraph with no cycles (i.e., $F \subseteq E$)
- \mathcal{F} : set of all forests $\rightarrow (E, \mathcal{F})$ is a matroid
- Greedy algorithm gives maximum weight forest (equivalent to MST problem)

Bicircular matroid of a graph $G = (V, E)$:

- \mathcal{B} : set of edges such that every connected subset has ≤ 1 cycle
- (E, \mathcal{B}) is a matroid \rightarrow greedy gives max. weight such subgraph

Linearly independent vectors:

- Vector space V , E : finite set of vectors, I : sets of lin. indep. vect.
- Fano matroid can be defined like that

Forest Matroid



E : edge set of a graph I : all subsets of E with no cycles

Greedoid

- Matroids can be generalized even more

- Relax hereditary property:

Replace $A' \subseteq A \in I \implies A' \in I$

by $\emptyset \neq A \in I \implies \exists a \in A, \text{ s.t. } A \setminus \{a\} \in I$

- Exchange property holds as before
- Under certain conditions on the weights, greedy is optimal for computing the max. weight $A \in I$ of a greedoid.
 - Additional conditions automatically satisfied by hereditary property
- More general than matroids