Some Notes on Resolution Calculus

1. Ask the correct Question: Usually we are interested in the question whether $KB \models \phi$ which is equivalent to asking the question

$$KB \cup \{\neg \phi\} \vdash_{\text{Res}} \bot.$$  

(to check whether $\phi$ is a tautology you first have to transform the question to the above format)

2. Transform the above question (1) into the correct CNF format. Let's look at a simple example to see how to do it.

$$KB = \{A \lor B, \neg B \lor C, C \lor A\} \quad (2)$$
$$\phi = A \land B \quad (3)$$

Now transforming

$$KB \cup \{\neg \phi\} = \{A \lor B, \neg B \lor C, C \lor A, \neg \phi\} \quad (4)$$
$$= \{A \lor B, \neg B \lor C, C \lor A, \neg A \lor \neg B\} \quad (5)$$

So the CNF is

$$CNF = (A \lor B) \land (\neg B \lor C) \land (C \lor A) \land (\neg A \lor \neg B) \quad (6)$$

(Note that in this step you might have to do more conversions to obtain a CNF. This was a simple example. But you should always append $\{\neg \phi\}$ with a $\land$ and not with a $\lor$.)

3. Transform the CNF back to the set notation.

$$\{A \lor B, \neg B \lor C, C \lor A, \neg A \lor \neg B\}$$

4. Try to deduce the empty clause $\Box$ with the inference rule, if you can deduce it your answer will be yes otherwise it will be no (Your answer is only worth something because the Resolution Calculus is refutation-complete).

Note that the $\cup$ in $KB \cup \{\neg \phi\}$ equals to $\land$ (and not to $\lor$) because you just have to add $\{\neg \phi\}$ as another formula to your knowledge base.

However, when you try to understand the inference rule of resolution calculus

$$R : \frac{C_1 \cup \{L\}, C_2 \cup \{\neg L\}}{C_1 \cup C_2}$$

the union $\cup$ is a $\lor$. The difference that this $\cup$ here is on the level of clauses which are concatenated by $\lor$. The $\cup$ above is on the level of formulas in the knowledge base and formulas in the knowledge base are concatenated by $\land$. 

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