# Some Notes on Resolution Calculus

1. Ask the correct Question: Usually we are interested in the question whether $KB \models \phi$ which is equivalent to asking the question

$$KB \cup \{\neg\varphi\} \vdash_{\textbf{Res}} \bot. \tag{1}$$

   (to check whether $\phi$ is a tautology you first have to transform the question to the above format)

2. Transform the above question (1) into the correct CNF format. Let's look at a simple example to see how to do it.

$$KB = \{A \vee B, \neg B \vee C, C \vee A\} \tag{2}$$
$$\phi = A \wedge B \tag{3}$$

   Now transforming

$$KB \cup \{\neg\varphi\} = \{A \vee B, \neg B \vee C, C \vee A, \neg\phi\} \tag{4}$$
$$= \{A \vee B, \neg B \vee C, C \vee A, \neg A \vee \neg B\} \tag{5}$$

   So the CNF is

$$CNF = (A \vee B) \wedge (\neg B \vee C) \wedge (C \vee A) \wedge (\neg A \vee \neg B) \tag{6}$$

   (Note that in this step you might have to do more conversions to obtain a CNF. This was a simple example. But you should always append $\{\neg\varphi\}$ with a $\wedge$ and not with a $\vee$.)

3. Transform the CNF back to the set notation.

$$\{A \vee B, \neg B \vee C, C \vee A, \neg A \vee \neg B\}$$

4. Try to deduce the empty clause $\square$ with the inference rule, if you can deduce it your answer will be *yes* otherwise it will be *no* (Your answer is only worth something because the Resolution Calculus is refutation-complete).

Note that the $\cup$ in $KB \cup \{\neg\varphi\}$ equals to $\wedge$ (and not to $\vee$) because you just have to add $\{\neg\varphi\}$ as another formula to your knowledge base.

However, when you try to understand the inference rule of resolution calculus

$$\mathbf{R}: \quad \frac{C_1 \cup \{L\}, C_2 \cup \{\neg L\}}{C_1 \cup C_2}.$$

the union $\cup$ is a $\vee$. The difference that this $\cup$ here is on the level of *clauses* which are concatenated by $\vee$. The $\cup$ above is on the level of formulas in the knowledge base and formulas in the knowledge base are concatenated by $\wedge$.