Exercise 1: The Empty String (3 points)

The empty string $\varepsilon$ is defined as a string of length $|\varepsilon| = 0$. Shortly explain the difference between $\varepsilon$, $L_1 := \emptyset$ and $L_2 := \{\varepsilon\}$. Construct a corresponding DFA where applicable.

Exercise 2: Constructing DFAs (1+2+3 points)

Let $\Sigma = \{0, 1\}$ be an alphabet. Construct DFAs that recognize the following languages over $\Sigma$.

(a) $L_1 = \{w \in \Sigma^* \mid w \text{ contains at most one } 1\}$.

(b) $L_2 = \{w \in \Sigma^* \mid w \text{ contains the substring } 00\}$.

(c) $L_1 \cup L_2$.

Remark: $\Sigma^* := \{a_1 \cdots a_n \mid a_i \in \Sigma, n \in \mathbb{N}_0\}$ is the set of all strings over the alphabet $\Sigma$.

Exercise 3: Closure under Complement and Intersection (2+3 points)

Let $L, L_1, L_2$ be regular languages. Show that both $L := \Sigma^* \setminus L$ and $L_1 \cap L_2$ are regular as well.

Exercise 4: NFA to DFA Conversion (2+4 points)

Consider the following NFA.

(a) Give a formal description of the above NFA, i.e. $(Q, \Sigma, \delta, q_0, F)$. Write down the transition function $\delta$ explicitly (e.g. in the form $\delta(q, x) = A$ for any transition from $q \in Q$ to $A \subseteq Q$ with the symbol $x \in \Sigma \cup \{\varepsilon\}$).

(b) Construct a DFA which is equivalent to the above NFA.