Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2016
Exercise Sheet 3

Hand in (electronically or hard copy) before your weekly meeting but not later than 23:59, Wednesday, November 16, 2016

Exercise 1: Regular Languages in all Shapes and Sizes (2+2+2 points)

The following regular languages can be denoted explicitly as a set of words (cf. (c)), as regular expressions or as finite automatons. Give the two missing notations of the following representations of regular languages.

(a) The language represented by the automaton given in Figure 1.

(b) $a^*(ab \cup ba)b^*$

(c) $\{w_1 \cdots w_n \mid w_i \in \{a,b\}^*, |w_i| \leq 4, w_i$ contains equal number of a's and b's$, n \geq 1\}$
Note that we restated this exercise to eliminate some ambiguity.

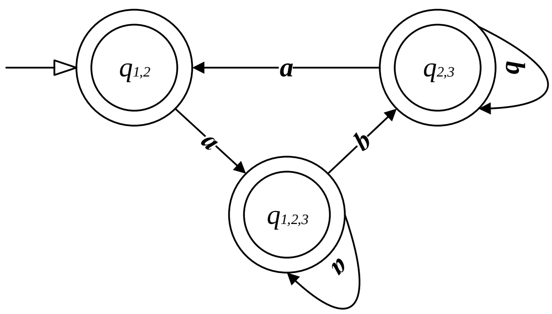

Figure 1: Finite automatom of a regular language.

Exercise 2: Limitations of the Pumping Lemma (3+3+1 points)

Let Σ be an alphabet. Consider the *Pumping Lemma* in the following notation:

$$L \subseteq \Sigma^* \text{ regular} \implies \exists\, p \in \mathbb{N}\ \forall\, s \in \{w \in L \mid |w| \geq p\}\ \exists\, x,y,z \in \Sigma^*\ \ s = xyz \text{ and}$$
$$(1)\ \forall i \in \mathbb{N}_0\ \ xy^iz \in L \text{ and}$$
$$(2)\ |y| > 0 \text{ and}$$
$$(3)\ |xy| \leq p$$

Remark: The right hand side of the $\Rightarrow$ symbol says that any word of a given language L longer than p can be 'pumped' resulting in a word that is contained in L again.

(a) Show that for a given, fixed $c \in \mathbb{N}$ the language $\{w \in \Sigma^* \mid |w| \leq c\}$ is regular. Does your result conflict with the Pumping Lemma?

(b) The right hand side of the Pumping Lemma is not a sufficient condition for regularity of any given language L! Show this by giving a counterexample. You can use that the language $\{c^m a^n b^n \mid n, m \geq 1\} \cup \{a, b\}^*$ is not regular.

(c) Give a Venn-Diagram showing the relation between the set of all languages over Σ, the set of regular languages over Σ and the set of languages over Σ for which the right hand side of the Pumping Lemma holds.

Exercise 3: Applications of the Pumping Lemma (2+2+3 points)

State the *contraposition* of the Pumping Lemma given in the previous exercise.

Hint: An implication $A \Rightarrow B$ *is logically equivalent to its contraposition* $\neg B \Rightarrow \neg A$*. For the negation of quantified expressions the following holds:* $\neg \forall x : P(x) \Leftrightarrow \exists x : \neg P(x)$ *and* $\neg \exists x : P(x) \Leftrightarrow \forall x : \neg P(x)$*.*

Show that the following languages are not regular using the contraposition of the Pumping Lemma.

(a) $L_1 := \{a^n b^n \mid n \geq 0\}$.

(b) $L_2 := \{w w^R \mid w \in \{a, b\}^*\}$ where w^R is defined as w in reverse order.

(c) $L_3 := \{a^q \mid q \text{ is prime}\}$