Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2016
Exercise Sheet 7

Hand in (electronically or hard copy) before your weekly meeting but not later than 23:59, Wednesday, December 14, 2016

Exercise 1: The Halting Problem (2+2 points)

A Turing machine *recognizes* a language L over the alphabet Σ, if it halts in the accepting state if and only if $s \in L$. Thus the set of *Turing recognizable* languages is the same as the set of *Recursive Enumerable* languages RE from the previous exercise. Commonly $L \in \text{RE}$ is also called *semi-decidable*. If a Turing machine recognizes L and halts for *every* input string $s \in \Sigma^*$ then it *decides* L and L is called *decidable*. If no *decider* exists for a language L, then L is called *undecidable*.

(a) Show that the Halting problem $H := \{\langle M, s\rangle \mid \text{Turing machine } M \textit{ halts} \text{ on input } s\}$ is semi-decidable[1].

(b) Show that the Halting problem H is undecidable.
Hint: You may use that we know that $U := \{\langle M, s\rangle \mid \text{Turing machine } M \textit{ accepts} \text{ input } s\}$ *is an undecidable language from the lecture.*

Exercise 2: Language Classes and Set Operations (2+3+1 points)

(a) Show that RE is not closed under complementation. I.e. for $L \in \text{RE}$, $\overline{L} := \Sigma^* \setminus L$ is in general not semi-decidable.
Hint: Use that with the results from exercise 1 you have a language which is undecidable but semi-decidable. What if $\overline{H}$ *were also semi-decidable?*

(b) Argue why the set of decidable languages is closed under union, intersection and complement.

(c) Give a table showing whether or not the set of regular, context-free, decidable, semi-decidable languages are *in general* closed under the operations union, intersection and complement.

Exercise 3: Relation between Language Classes (2+2+2 points)

(a) Give a Venn diagram showing the relation between the set of regular, context-free, decidable and semi-decidable languages.

(b) Give an explanation why some of these sets are contained in others.
Hint: Argue with the according automaton models that represent these language classes.

(c) Show that the subset relations are proper, e.g. by giving a language which is contained in the respective superset but not in the subset.
Hint: You can cite results from previous exercises.

[1] Our definition of the halting problem deviates from the one on the lecture slides, but is also very common.

Exercise 4: Decidable Languages (2+2+0 points)

(a) Show that $\{\langle A\rangle \mid A \text{ is a CFG that can generate } \varepsilon\}$ is decidable.

(b) Show that $\{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ is decidable.

(c) Show that $\{\langle R, S\rangle \mid R, S \text{ are regular expressions and } L(R) \subseteq L(S)\}$ is decidable. (This is a voluntary exercise, no points will be rewarded.)