Exercise 1: Induction \((6 \text{ Points})\)

Prove the following summation formula by induction on \(n\):

For all natural numbers \(n \geq 1\) it holds:

\[
\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)
\]

Exercise 2: Sets & More \((8 \text{ Points})\)

Either proof the following statements or show that they are not true.

1. \(\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x + y = 0\} \subseteq \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \cdot y = 0\}\)

2. \(A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)\)

3. \(A \subseteq B\) implies \(C \cup A \subsetneq C \cup B\).

Exercise 3: Where is the even degree node? \((6 \text{ Points})\)

A simple graph is a graph without self loops, i.e., every edge of the graph is an edge between two distinct nodes. The degree \(d(v)\) of a node \(v \in V\) in an undirected graph \(G = (V, E)\) is the number of its neighbors, i.e,

\[
d(v) = |\{u \in V \mid \{v, u\} \in E\}|
\]

Show that every simple graph with an odd number of nodes contains a node with even degree.

Hint: Consider the sum \(D = \sum_{v \in V} d(v)\) of all degrees. Is \(D\) odd or even?