Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 4

Hand in (electronically or hard copy) by 12:15 pm, November 20th, 2017

Exercise 1: Constructing Pushdown Automata *(6 Points)*

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$.
Construct a PDA $\mathcal{A}$ with $L(\mathcal{A}) = L$.

Exercise 2: Understanding PDAs *(4 Points)*

Consider the PDA $\mathcal{A} = (\{q_0, q_1, q_2\}, \{a, b\}, \{\$, Z\}, q_0, \delta, \{q_2\})$ with the following transition relation δ

$$(q_0, a, \$) \mapsto \{(ZZ\$, q_0)\} \qquad (q_0, a, Z) \mapsto \{(ZZZ, q_0)\}$$
$$(q_0, b, Z) \mapsto \{(\epsilon, q_1)\} \qquad (q_1, b, Z) \mapsto \{(\epsilon, q_1)\}$$
$$(q_1, \epsilon, \$) \mapsto \{(\epsilon, q_2)\}$$

Remark: Assume that the stack contains the symbol \$ at the start.

1. Decide which of the words $b, aabbbb$ and $abbb$ are accepted by $\mathcal{A}$. Explain your answers by either giving an accepting sequence of configurations or by explaining why non sequence of configurations is accepting.
2. Which language is recognized by $\mathcal{A}$?

Exercise 3: Context Free Grammar *(6 Points)*

Give a contextfree grammar for each of the following languages.

1. $L_1 = \{a^k b^{3k} | k \geq 0\}$
2. $L_2 = \{a^i b^j | 0 < i \leq j\}$
3. $L_1 \cdot L_2$
4. $L_1 \cup L_2$

Exercise 4: Chomsky Normal Form. *(4 Points)*

Convert the following grammar into Chomsky normal form along the procedure given in the lecture.

$$\begin{aligned} S &\rightarrow AB \mid A \mid B \\ A &\rightarrow aAA \mid aA \mid a \\ B &\rightarrow bBB \mid bB \mid b \end{aligned}$$

It is **not** sufficient to just state the final grammar without intermediate steps.
Which language is recognized by the grammar?