Exercise 1: Constructing Pushdown Automata

Consider the language \( L = \{a^m b^{2m} ba^n \mid m, n > 0\} \) over the alphabet \( \Sigma = \{a, b\} \).
Construct a PDA \( A \) with \( L(A) = L \).

Exercise 2: Understanding PDAs

Consider the PDA \( A = (\{q_0, q_1, q_2\}, \{a, b\}, \{\$, Z\}, q_0, \delta, \{q_2\}) \) with the following transition relation \( \delta \):
\[
\begin{align*}
(q_0, a, \$) &\rightarrow \{(ZZ\$, q_0)\} \\
(q_0, a, Z) &\rightarrow \{(ZZZ, q_0)\} \\
(q_0, b, Z) &\rightarrow \{\epsilon, q_1\} \\
(q_1, \epsilon, \$) &\rightarrow \{\epsilon, q_2\}
\end{align*}
\]
Remark: Assume that the stack contains the symbol \( \$ \) at the start.

1. Decide which of the words \( b, aabbbb \) and \( abbb \) are accepted by \( A \). Explain your answers by either giving an accepting sequence of configurations or by explaining why non sequence of configurations is accepting.

2. Which language is recognized by \( A \)?

Exercise 3: Context Free Grammar

Give a contextfree grammar for each of the following languages.

1. \( L_1 = \{a^k b^{2k} \mid k \geq 0\} \)
2. \( L_2 = \{a^i b^j \mid 0 < i \leq j\} \)
3. \( L_1 \cdot L_2 \)
4. \( L_1 \cup L_2 \)

Exercise 4: Chomsky Normal Form.

Convert the following grammar into Chomsky normal form along the procedure given in the lecture.
\[
S \rightarrow AB \mid A \mid B \\
A \rightarrow aAA \mid aA \mid a \\
B \rightarrow bBB \mid bB \mid b
\]
It is not sufficient to just state the final grammar without intermediate steps.
Which language is recognized by the grammar?