Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 7

Hand in (electronically or hard copy) by 12:15 pm, December 11th, 2017

Exercise 1: Decidability

Consider the following language

\[ A_{\text{all}} = \{ \langle M \rangle \mid \text{TM } M \text{ accepts all inputs} \} \]

Show that \( A_{\text{all}} \) is not decidable.

**Hint:** Assume that a TM \( M_c \) decides \( A_{\text{all}} \). Then run the Turing machine \( M_c \) on some very wisely chosen input to decide the halting problem.

Exercise 2: Landau Notation

Prove or disprove the following statements

(a) \( 100\sqrt{n} \in O(0.001 \cdot n) \).
(b) \( \log_2 3^n \in O(n) \).
(c) \( 2 \cdot n \in O(10 \cdot \sqrt{n}) \).

**Remark:** There are thousands of similar exercises on the 'net. Go through some of them to practice for the exam.

Exercise 3: Sort Functions by Asymptotic Growth

Sort the following functions by asymptotic growth using the \( O \)-notation. Write \( g <_O f \) if \( g \in O(f) \) and \( g \notin O(f) \). Write \( g =_O f \) if \( f \in O(g) \) and \( g \in O(f) \).

\[
\begin{array}{cccc}
n^2 & \sqrt{n} & 2^n & \log(n^2) \\
3^n & n^{100} & \log(\sqrt{n}) & (\log n)^2 \\
\log n & 10^{100}n & n! & n \log n \\
n \cdot 2^n & n^n & \sqrt{\log n} & n \\
\end{array}
\]