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Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 3

Hand in (electronically or hard copy) by 12:15 pm, November 13th, 2017

Exercise 1: Regular Expressions *(4 Points)*

Consider the regular expression $r = (aa^* + ba)^* \cdot (bbb + ab^*) + (ab)^*$. State for each of the following words whether they are contained in $L(r)$

$$\epsilon, baabbb, bbbb, abab, ababa, ababaa, a.$$

In case that a word is contained in $L(r)$ show how to obtain the word from r, e.g., by marking the corresponding parts in r.

Sample Solution

- ϵ: $(aa^* + ba)^* \cdot (bbb + ab^*) + \underline{(ab)^*}$
- $baabbb$: $(aa^* + \underline{ba})^* \cdot (bbb + \underline{ab^*}) + (ab)^*$
- $bbbb$: The word is not contained in the language as using $(aa^* + ba)^*$ or $(ab)^*$ implies that the word is empty or contains an a and $(bbb + ab^*)$ cannot produce four consecuitive bs.
- $abab$: $(aa^* + ba)^* \cdot (bbb + ab^*) + \underline{(ab)^*}$
- $ababa$: The string is not conained in the language. The word cannot be created with $(ab)^*$ as all words created with $(ab)^*$ end with a b.

  The word can also not created with $(aa^* + ba)^* \cdot (bbb + ab^*)$: All words created with that regular expression have to use the part $(bbb + ab^*)$ for a single time at the end of the word, which for $ababa$ implies that $(bbb + ab^*)$ would have to generate an a (the expression cannot generate ba or any longer substring of the word ending at the end).

  This means that we would have to generate $abab$ with $(aa^* + ba)^*$ but all strings generated by that expression cannot end with a b.
- $ababaa$:$\underline{(\underline{a}a^* + \underline{ba})^*} \cdot (bbb + \underline{a}b^*) + (ab)^*$
- a: $(aa^* + ba)^* \cdot (bbb + \underline{a}b^*) + (ab)^*$.

Exercise 2: Regular Expressions 2 *(6 Points)*

1. Let L_1 be the language consisting of words of the form $w_1 w_2 w_3$ with $w_1, w_2, w_3 \in \{a, b, c\}^*$ **and** w_1 contains **no** a's **and** w_2 contains **no** b's **and** w_3 contains **no** c's.

   Give a regular expression that generates L_1.

2. Let $L_2 \subseteq \{a, b\}^*$ be the language of all words that do not have any of the words $\{aaa, aaaa, \ldots\}$ as a consecutive substring.

   Give a regular expression that generates L_2.

Sample Solution

1. $(b+c)^* \cdot (a+c)^* \cdot (a+b)^*$.
2. $b^* + (b^*(a+aa)b^+)^*(a+aa)b^*$ or alternatively $(\epsilon + a + aa)(b + ba + baa)^*$.

Exercise 3: Pumping Lemma *(6 Points)*

Use the pumping lemma to show that $L = \{a^i b^j \mid i \neq j\}$ is not regular.

Sample Solution

We prove that L is not regular by showing that L violates the pumping condition, i.e. always contains a string that can not be pumped. Let p be the pumping length. We investigate the string $s = a^p b^{p+p!} \in L$. Furthermore s is longer than p.
Consider a partition $xyz = s$ with $|y| \geq 1$ and $|xy| \leq p$. It follows that $y = a^q$ with $1 \leq q \leq p$. Set $k = p!/q + 1 \in \mathbb{N}$. Then we have that $xy^k z = a^{p+(k-1)q} b^{p+p!} = a^{p+p!} b^{p+p!} \notin L$. Regardless of the partition we gave a pumped string that is not contained in the language.
Remark: This solution was pretty tough. You should also look at exercises from previous years to find easier pumping lemma questions.

Exercise 4: Context Free Grammar *(4 Points)*

Give a context free grammar for the language $L = \{a^i b^j \mid i \neq j\}$.

Sample Solution

S is the start symbol.

$$S \to AaTb \mid aTbB \mid A \mid B$$
$$T \to \epsilon \mid aTb$$
$$A \to a \mid aA$$
$$B \to b \mid bB.$$