Albert-Ludwigs-Universität, Inst. für Informatik
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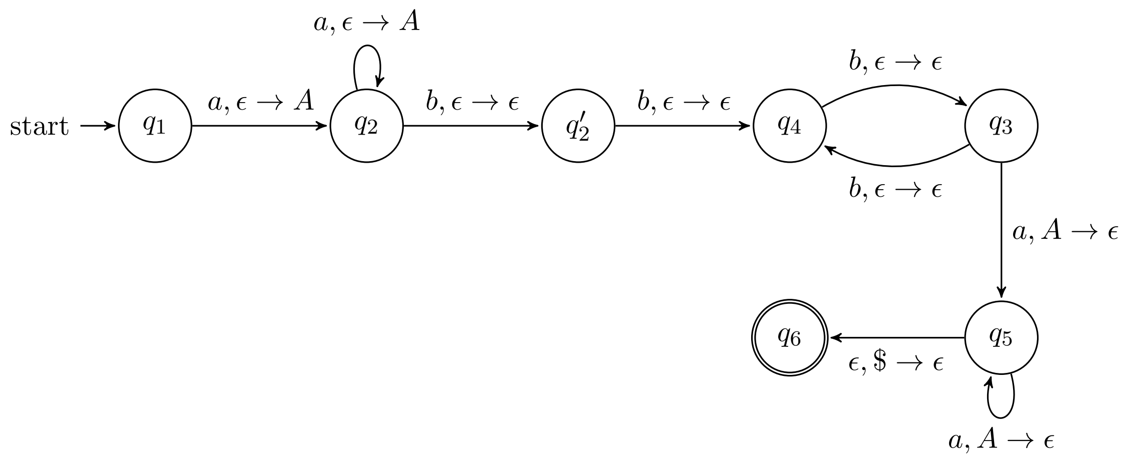
Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 4

Hand in (electronically or hard copy) by 12:15 pm, November 20th, 2017

Exercise 1: Constructing Pushdown Automata *(6 Points)*

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$.
Construct a PDA $\mathcal{A}$ with $L(\mathcal{A}) = L$.

Sample Solution



The formal definition of the automaton is implicitly given.

Exercise 2: Understanding PDAs *(4 Points)*

Consider the PDA $\mathcal{A} = (\{q_0, q_1, q_2\}, \{a, b\}, \{\$, Z\}, q_0, \delta, \{q_2\})$ with the following transition relation δ

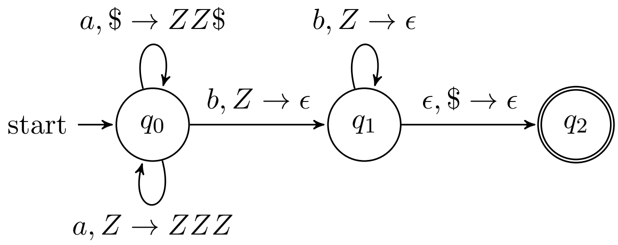
$$(q_0, a, \$) \mapsto \{(ZZ\$, q_0)\} \qquad (q_0, a, Z) \mapsto \{(ZZZ, q_0)\}$$
$$(q_0, b, Z) \mapsto \{(\epsilon, q_1)\} \qquad (q_1, b, Z) \mapsto \{(\epsilon, q_1)\}$$
$$(q_1, \epsilon, \$) \mapsto \{(\epsilon, q_2)\}$$

Remark: Assume that the stack contains the symbol \$ at the start.

1. Decide which of the words $b, aabbbb$ and $abbb$ are accepted by $\mathcal{A}$. Explain your answers by either giving an accepting sequence of configurations or by explaining why non sequence of configurations is accepting.
2. Which language is recognized by $\mathcal{A}$?

Sample Solution

The automaton looks as follows.



1. It is clear that the word b cannot be accepted as a b is only processed by the automaton if the top of the stack contains a Z and the stack can only contain a Z if an a has been read.

   The word $aabbbb$ is accepted by the language by the following sequence of configurations.

   $$(aabbbb, q_0, \$) \to (abbbb, q_0, ZZ\$) \to (bbbb, q_0, ZZZZ\$) \to (bbb, q_1, ZZZ\$) \\ \to (bb, q_1, ZZ\$) \to (b, q_1, Z\$) \to (\epsilon, q_1, \$) \to (\epsilon, q_2, \epsilon)$$

   The word $abbb$ is not accepted by the automaton. The only deterministicly ocurring sequence of configurations when reading the word is

   $$(abbb, q_0, \$) \to (bbb, q_0, ZZ\$) \to (bb, q_1, Z\$) \to (b, q_1, \$).$$

   In this last configuration there is no transition which can be executed and the state q_1 is not accepting.

2. The automaton recognizes the language $\{a^n b^{2n} \mid n \geq 1\}$

Exercise 3: Context Free Grammar *(6 Points)*

Give a contextfree grammar for each of the following languages.

1. $L_1 = \{a^k b^{3k} | k \geq 0\}$
2. $L_2 = \{a^i b^j | 0 < i \leq j\}$
3. $L_1 \cdot L_2$
4. $L_1 \cup L_2$

Sample Solution

S_1, S_2, T_1 and T_2 are the respective start symbols. The rest of the formal definition of the grammars is given implicitly.

1.

$$S_1 \to aS_1bbb \mid \epsilon$$

2.

$$\begin{aligned} S_2 &\to aRb \\ R &\to aRb \mid B \\ B &\to Bb \mid b \mid \epsilon \end{aligned}$$

3.

$$T_1 \to S_1S_2$$

4.

$$T_2 \to S_1 \mid S_2$$

Exercise 4: Chomsky Normal Form. *(4 Points)*

Convert the following grammar into Chomsky normal form along the procedure given in the lecture.

$$\begin{aligned} S &\to AB \mid A \mid B \\ A &\to aAA \mid aA \mid a \\ B &\to bBB \mid bB \mid b \end{aligned}$$

It is **not** sufficient to just state the final grammar without intermediate steps.
Which language is recognized by the grammar?

Sample Solution

The grammar recognized the language which corresponds to $L(a^*b^*) \setminus \{\epsilon\}$.
Remove rules of the form $S \to B$ according to the lecture and obtain.

$$\begin{aligned} S &\to AB \mid aAA \mid aA \mid a \mid bBB \mid bB \mid b \\ A &\to aAA \mid aA \mid a \\ B &\to bBB \mid bB \mid b \end{aligned}$$

Transform according to step four in the lecture:

$$\begin{aligned} S &\to AB \mid aAA \mid aA \mid a \mid bBB \mid bB \mid b \\ A &\to aT_1 \mid aA \mid a \\ B &\to bT_2 \mid bB \mid b \\ T_1 &\to AA \\ T_2 &\to BB \end{aligned}$$

Substitute terminal symbols (a and b) with new non-terminals (U_a and U_b) in terms of length two:

$$\begin{aligned} S &\to AB \mid U_aT_1 \mid U_aA \mid a \mid U_bT_2 \mid U_bB \mid b \\ A &\to U_aT_1 \mid U_aA \mid a \\ B &\to U_bT_2 \mid U_bB \mid b \\ T_1 &\to AA \\ T_2 &\to BB \\ U_a &\to a \\ U_b &\to b. \end{aligned}$$