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Algorithms Theory
Exercise Sheet 4

Due: Monday, 17th of December, 2018, 14:15 pm

Exercise 1: Priority Queues and Prim's Algorithm *(6+4 Points)*

(a) Assume we want to store $(key, data)$-pairs in a priority queue where the priorities (keys) are only from the set $\{1, \ldots, c\}$ and $c \in \mathbb{N}$ is constant.

Describe a priority queue that provides the operations `Insert`$(key, data)$, `Get-Min`, `Delete-Min`, and `Decrease-Key`$(pointer, newkey)$ all in constant time for the given scenario, and describe how these operations work on your data structure.

(b) State how fast Prim's algorithm to compute a minimum spanning tree is, under the assumption that edge weights are in the set $\{1, \ldots, c\}$ and $c \in \mathbb{N}$ is constant, using your implementation of a priority queue. Explain your answer.

***Remark**: Assume you have a priority queue as in (a), even if you did not succeed in (a).*

Exercise 2: Capacity Change in Flow Networks *(4+6 Points)*

We are given a maximum flow network $G = (V, E)$ with integer capacities together with a maximum flow Φ. Describe an algorithm with time complexity $O(|V| + |E|)$ to compute a new maximum flow for each of the following cases:

(a) if the capacity of an arbitrary edge $(u, v) \in E$ increases by one unit.

(b) if the capacity of an arbitrary edge $(u, v) \in E$ decreases by one unit.

Exercise 3: Fibonacci Heaps *(10 Points)*

Show that for any positive integer n, there exists a sequence of Fibonacci Heap operations that can construct a Fibonacci Heap consisting of just one tree that is a linear chain of n nodes. Provide the pseudocode of a recursive procedure to construct such a Fibonacci Heap, and show its correctness.

Exercise 4: *Edge Minimal* Minimum Cut *(4+6 Points)*

Consider an undirected, weighted graph $G = (V, E)$ with integral edge weights. Among all cuts of G with minimum weight you want to find a cut $(S, V \setminus S)$ with the smallest number of edges (i.e. edges with exactly one endpoint in S).

(a) Modify the weights of G to create a new graph G' in which any minimum cut in G' is a minimum cut with the smallest number of edges in G.

(b) Prove that G' has the property claimed in part (a).