Exercise 1: Induction \hspace{1cm} (7 Points)

Find a much more compact formula for the term $\sum_{k=1}^{n}(2k-1)$ and prove its correctness by induction.

\text{Hint: } \frac{n(n+1)}{2} \text{ would be such a formula for the expression } \sum_{k=1}^{n} k.

Exercise 2: Even Number of Odd Degree Nodes \hspace{1cm} (5 Points)

A \textit{simple graph} is a graph without self loops, i.e., every edge of the graph is an edge between two distinct nodes. The degree $d(v)$ of a node $v \in V$ of an undirected graph $G = (V, E)$ is the number of its neighbors, i.e,

$$d(v) = |\{u \in V \mid \{v, u\} \in E\}|.$$ 

Show that the number of nodes with odd degree in every simple graph is even.

\text{Hint: Consider the sum } D = \sum_{v \in V} d(v) \text{ of all degrees. Is } D \text{ odd or even?}

Exercise 3: Playing with Sets \hspace{1cm} (8 Points)

Let $A$ be a set. Show that the following three statements are equivalent.

(i) $B \setminus A = B$ for all sets $B$,

(ii) $(A \cup B) \setminus A = B$ for all sets $B$,

(iii) $A = \emptyset$.

\text{Hint: It is sufficient to prove that (i) } \Rightarrow \text{ (ii), (ii) } \Rightarrow \text{ (iii) and (iii) } \Rightarrow \text{ (i).}