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Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 1

for getting feedback submit (electronically) before the start of the tutorial on 29th of October 2018.

Exercise 1: Induction *(7 Points)*

Find a much more compact formula for the term $\sum_{k=1}^{n}(2k-1)$ and prove its correctness by induction.
Hint: $\frac{n(n+1)}{2}$ *would be such a formula for the expression* $\sum_{k=1}^{n} k$.

Exercise 2: Even Number of Odd Degree Nodes *(5 Points)*

A *simple graph* is a graph without self loops, i.e., every edge of the graph is an edge between two distinct nodes. The degree $d(v)$ of a node $v \in V$ of an undirected graph $G = (V, E)$ is the number of its neighbors, i.e,

$$d(v) = |\{u \in V \mid \{v, u\} \in E\}|.$$

Show that the number of nodes with odd degree in every simple graph is even.

Hint: Consider the sum $D = \sum_{v \in V} d(v)$ *of all degrees. Is* D *odd or even?*

Exercise 3: Playing with Sets *(8 Points)*

Let A be a set. Show that the following three statements are equivalent.

(i) $B \setminus A = B$ for all sets B,

(ii) $(A \cup B) \setminus A = B$ for all sets B,

(iii) $A = \emptyset$.

Hint: It is sufficient to prove that (i) $\Rightarrow$ *(ii), (ii)* $\Rightarrow$ *(iii) and (iii)* $\Rightarrow$ *(i).*