Exercise 1: Constructing DFAs

Construct DFAs that recognize the following languages. Drawing the state diagrams is sufficient. The alphabet is $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid |w| \geq 2 \text{ and } w \text{ contains an odd number of ones}\}$.

(b) $L_2 = \{w \mid w \text{ contains an even number of zeros}\}$.

(c) $L_3 = \{w \mid \text{in } w \text{ every zero is immediately followed by a one}\}$.

(d) $L_4 = \{w \mid w \text{ ends with 01}\}$.

Exercise 2: Maxstring

Let

$$maxstring(L) = \{w : w \in L \text{ and for all words } z \in \Sigma^* : z \neq \epsilon \Rightarrow wz \notin L\}.$$

(a) What is $maxstring(L_1 L_2)$, where $L_1 = \{w \in \{a, b\}^* : w \text{ contains exactly one } a\}$ and $L_2 = \{a\}$?

(b) Explain how to prove that the regular languages are closed under maxstring.

Hint: Let $L$ be a regular language. You need to prove that $maxstring(L)$ is regular as well.
Exercise 3: From NFA to DFA

Consider the following NFA.

(a) Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.

(b) Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

(c) Explain which language the automaton accepts.