

Theoretical Computer Science - Bridging Course

Summer Term 2018

Exercise Sheet 3

for getting feedback submit (electronically) before the start of the tutorial on
12th of November 2018.

Exercise 1: Regular Expressions

(2+6 Points)

- (a) Which of the following words are in the language described by the regular expression $r = a(ab)^*a$.
- (a) abababa
 - (b) aaba
 - (c) aabbaa
 - (d) aba
 - (e) aabababa
- (b) Give a regular expression for each of the following languages (you do not need to prove the correctness of your answers).
- (a) $L_1 = \{ab, ba, bb, aa, abab\}$.
 - (b) $L_2 = \{w_1w_2w_3 \in \{a, b, c\}^* \mid w_1 \text{ contains an even number of a's, } w_2 \text{ contains no } b, w_3 \in L_1\}$
 - (c) L_3 is the language over alphabet $\{a, b\}$ that consists of all words that do not contain the substring aa .

Exercise 2: The Pumping Lemma: Sufficiency or Necessity? (4 Points)

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

- (a) Describe in words (not using the pumping lemma), why L cannot be a regular language.
- (b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.
- Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.*

Exercise 3:

(6 Points)

Let $\Sigma = \{0, 1\}$, prove the following:

- (a) (3 points) The language $A = \{0^k w 0^k \mid k \geq 1 \text{ and } w \in \Sigma^*\}$ is regular.
- (b) (3 points) The language $B = \{0^k 1 w 0^k \mid k \geq 1 \text{ and } w \in \Sigma^*\}$ is not regular.

Exercise 4: Context Free Grammar

(9 Points)

Give context-free grammars that generate the following languages. The alphabet set is $\Sigma = \{0, 1\}$.

- (a) (2 points) $\{w \mid w \text{ starts and ends with the same symbol}\}$
- (b) (3 points) $\{w \mid \text{the length of } w \text{ is odd}\}$
- (c) (4 points) The complement of the language $\{0^n 1^n \mid n \geq 0\}$