

# Theoretical Computer Science - Bridging Course

## Summer Term 2018

### Exercise Sheet 4

for getting feedback submit (electronically) before the start of the tutorial on  
19th of November 2018.

#### Exercise 1: Constructing Pushdown Automata (6 Points)

Consider the language  $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$  over the alphabet  $\Sigma = \{a, b\}$ .  
Construct a PDA  $\mathcal{A}$  with  $L(\mathcal{A}) = L$ .

#### Exercise 2: Chomsky Normal Form (5 Points)

Use the algorithm from the lecture to give a grammar in Chomsky Normal Form that generates the same language as the grammar  $G = (V; \Sigma; R; S)$  with  $V = \{S; X; Y\}$ ,  $\Sigma = \{a, ab, c\}$ , and  $R$  being the following set of rules:

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow abb|aXb|\epsilon \\ Y &\rightarrow c|cY \end{aligned}$$

#### Exercise 3: Context-Free Languages and Set Operations (3+3 Points)

- (a) Show that context-free languages are not closed under taking intersections (i.e., the intersection of two context-free languages is not necessarily context free).  
*Hint: You can use that the language  $\{a^i b^i c^i \mid i \geq 0\}$  is not context-free.*
- (b) Show that context-free languages are not closed under taking complements.  
*Hint: You can use DeMorgan's law and the fact that the set of context-free languages is closed under performing union operations.*