

# Theoretical Computer Science - Bridging Course

## Summer Term 2018

### Exercise Sheet 5

for getting feedback submit (electronically) before the start of the tutorial on  
26th of November 2018.

#### Exercise 1: Constructing a Turing Machine (3 Points)

Consider alphabet  $A = \{1, 2, \dots, 9\}$ . We call a string  $S$  over  $A$  a *blue* string, if and only if the string consisting of the odd-positioned symbols in  $S$  is the reverse of the string consisting of the even-positioned symbols in  $S$ . For example  $S = 14233241$  is a blue string since the substring of the odd-positioned symbols is 1234 which is the reverse of the substring of the even-positioned symbols, i.e., 4321.

Design a Turing machine which accepts all blue strings over  $A$ . You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

#### Exercise 2: (4+2+2 Points)

- (a) Design a Turing Machine that decides the language  $L := \{0^n 1^n \mid n \geq 1\}$ . Explain your choice (you are supposed to explicitly construct the Turing machine).
- (b) Give the sequence of configurations of your Turing machine run on the string 0011.
- (c) Give the sequence of configurations of your Turing machine run on the string 0010.

*Remark: Here, you need to solve part a) to solve part b) and c). We would try to avoid such exercises in the exam.*

#### Exercise 3: Random Questions (2+2 Points)

- (a) Does the fact that the Halting Problem is not decidable mean that we can never tell if a program we have written is going to halt? Explain.
- (b) Describe how a Turing machine with arbitrary tape alphabet  $\Gamma_0$  can be simulated by a Turing machine with tape alphabet  $\Gamma_1 = \{0, 1, \square\}$  that never writes the symbol  $\square$  on the tape.

## Exercise 4: PDA to Turing Machine

(10 Points)

Let a  $k$ -PDA be a pushdown automaton that has  $k$  stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

- (a) (5 points) Show that 2-PDAs are more powerful than 1-PDAs. *Hint: Find a suitable language that cannot be recognized by a 1-PDA but can be recognized by a 2-PDA*
- (b) (5 points) Show that 3-PDAs are not more powerful than 2-PDAs. *Hint: Simulate a Turing machine tape with two stacks.*