Exercise 1: Constructing a Turing Machine

(3 Points)

Consider alphabet $A = \{1, 2, \ldots, 9\}$. We call a string $S$ over $A$ a blue string, if and only if the string consisting of the odd-positioned symbols in $S$ is the reverse of the string consisting of the even-positioned symbols in $S$. For example $S = 14233241$ is a blue string since the substring of the odd-positioned symbols is $1234$ which is the reverse of the substring of the even-positioned symbols, i.e., $4321$.

Design a Turing machine which accepts all blue strings over $A$. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

Exercise 2:

(4+2+2 Points)

(a) Design a Turing Machine that decides the language $L := \{0^n1^n \mid n \geq 1\}$. Explain your choice (you are supposed to explicitly construct the Turing machine).

(b) Give the sequence of configurations of your Turing machine run on the string $0011$.

(c) Give the sequence of configurations of your Turing machine run on the string $0010$.

Remark: Here, you need to solve part a) to solve part b) and c). We would try to avoid such exercises in the exam.

Exercise 3: Random Questions

(2+2 Points)

(a) Does the fact that the Halting Problem is not decidable mean that we can never tell if a program we have written is going to halt? Explain.

(b) Describe how a Turing machine with arbitrary tape alphabet $\Gamma_0$ can be simulated by a Turing machine with tape alphabet $\Gamma_1 = \{0, 1, \Box\}$ that never writes the symbol $\Box$ on the tape.
Exercise 4: PDA to Turing Machine \textit{(10 Points)}

Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

(a) (5 points) Show that 2-PDAs are more powerful than 1-PDAs. \textit{Hint: Find a suitable language that cannot be recognized by a 1-PDA but can be recognized by a 2-PDA}

(b) (5 points) Show that 3-PDAs are not more powerful than 2-PDAs. \textit{Hint: Simulate a Turing machine tape with two stacks.}