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Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 7

for getting feedback submit (electronically) before the start of the tutorial on 10th of December 2018.

Exercise 1: Decidability *(5+5 Points)*

(a) Is the following language decidable? Either prove that it is not decidable or provide an algorithm that decides it.

INDEPENDENT SET $=\{\langle G,k\rangle \mid G$ is a graph and contains an independent set of size $k\}$.

Remark: An independent set of a graph with size s is a set $S \subseteq V$, $|S| = s$ such that $\{v,w\} \notin E$ for all $u,w \in S$ and $|S|$ is its size..

(b) Let H be the language of the halting problem. Give a language L such that $L \cap H$ decidable and give a language K such that $K \cap H$ is undecidable. Prove your claims.

Exercise 2: $\mathcal{O}$-Notation Formal Proofs *(2+2+2 Points)*

The set $\mathcal{O}(f)$ contains all functions that are asymptotically not growing faster than the function f (when additive or multiplicative constants are neglected). That is:

$$g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)$$

For the following pairs of functions, check whether $f \in \mathcal{O}(g)$ or $g \in \mathcal{O}(f)$ or both. Proof your claims (you do not have to prove a negative result $\notin$, though).

(a) $f(n) = 100n,\ g(n) = 0.1 \cdot n^2$

(b) $f(n) = \log_2(n!),\ g(n) = n \log_2 n$ [*Hint:* $n! := \prod_{i=1}^{n} i \geq (n/2)^{n/2}$]

(c) $f(n) = 2^n,\ g(n) = 3^n$

Rermark: It is easy to produce tons of exercises of this type. Create a few exercises and try to solve them to practice this for the exam!

Exercise 3: Sort Functions by Asymptotic Growth *(6 Points)*

Sort the following functions by asymptotic growth using the $\mathcal{O}$-notation. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $f \notin \mathcal{O}(g)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

n^2	$\sqrt{n}$	2^n	$\log(n^2)$
3^n	n^{100}	$\log(\sqrt{n})$	$(\log n)^2$
$\log n$	$10^{100}n$	$n!$	$n \log n$
$n \cdot 2^n$	n^n	$\sqrt{\log n}$	n