Albert-Ludwigs-Universität
Institut für Informatik
Prof. Dr. F. Kuhn

Exam Theoretical Computer Science - Bridging Course

Tuesday, February 21, 2017, 9:00-10:30

Name: ....

Matriculation Nr.: ....

Signature: ....

Do not open or turn until told so by the supervisor!

- Write your **name** and **matriculation number** on this page and sign the document!
- Write your name on **all sheets**!
- Your **signature** confirms that you have answered all exam questions without any help, and that you have notified exam supervision of any interference.
- Write legibly and only use a pen (ink or ball point). Do **not use red**! Do **not use a pencil**!
- This is an **open book exam** therefore printed or hand-written material is allowed.
- However, **no electronic devices** are allowed.
- There are **seven tasks** (with several sub-tasks each) and there is a **total of 90 points**.
- **40 points are sufficient** in order to pass the exam.
- Only **one solution per task** is considered! Make sure to strike out alternative solutions, otherwise the one yielding the minimal number of points is considered.
- **Detailed steps** might help you to get more points in case your final result is incorrect.
- The keywords **Show...** or **Prove...** indicate that you need to prove or explain your answer carefully.
- The keywords **Give...** or **State...** indicate that you only need to provide a plain answer.
- **Read each task thoroughly** and make sure you understood what is required of you.

Task	Max. Points	Achieved
1	11	
2	14	
3	10	
4	13	
5	15	
6	12	
7	15	
Σ	90	

Task 1: Regular Languages (11 Points)

Let $L = \{ab(ab)^n b^m \mid n, m \in \mathbb{N} \cup \{0\}\}$ be a language over the alphabet $\Sigma = \{a, b\}$.

(a) Give a **regular expression** that generates L *(1 point)*.

(b) Give a **nondeterministic** finite automaton that recognizes L and has **at most three** states *(5 points)*.

(c) Give a **deterministic** finite automaton that recognizes L *(5 points)*.
Remark: You can directly give a deterministic automaton. No intermediate steps required.

Task 2: Context-Free Languages (14 Points)

Consider the language $L = \{(ab)^n(cd)^n \mid n \in \mathbb{N}\}$ over alphabet $\Sigma = \{a, b, c, d\}$. Note that $0 \notin \mathbb{N}$!

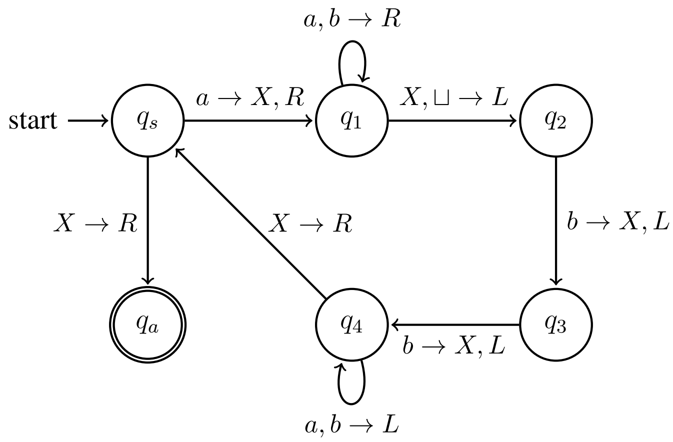
(a) Give a **context-free grammar** that generates L *(2 points)*.

(b) Give a grammar in **Chomsky Normal Form** (CNF) that generates L *(5 points)*.
Remark: You can directly give a grammar in CNF for L, no intermediate steps required.

(c) Prove that L is **not** a regular language. Use the Pumping Lemma *(7 points)*.

Task 3: Turing Machines (10 Points)

Consider the Turing machine M over the alphabet $\Sigma = \{a, b\}$, which is given via the following state diagram. *Note: The blank symbol $\sqcup \in \Gamma$ represents an empty cell on the tape.*



(a) Simulate M with input $s_1 = ab$ on its tape until it halts. Give **all configurations** that M passes through. State whether $s_1 \in L$ or not *(3 points)*.

(b) Simulate M with input $s_2 = abb$ on its tape until it halts. Give **all configurations** that M passes through. State whether $s_2 \in L$ or not *(4 points)*.

(c) Give a description of the language $L(M)$ that M recognizes in the **form of a set** *(3 points)*.

Task 4: $\mathcal{O}$ - Notation (13 Points)

State whether the following claims are true or false *(1 point each)*. Then **prove or disprove** the claim *(5 points for (a) and 6 points for (b))*. Use the definition of the $\mathcal{O}$-notation.

(a) $\log_2(2^n \cdot n^3) \in \mathcal{O}(n)$. ***Hint**: You may use that* $\log_2 n \leq n$ *for all* $n \in \mathbb{N}$.

(b) $\sqrt[3]{n^2} \in \mathcal{O}(\sqrt{n})$.

Task 5: Decidability (15 Points)

(a) Consider the problem VERTEXCOVER:

$$\textsc{VertexCover} := \{\langle G, k\rangle \mid \text{undirected, simple graph } G \text{ has a } \textbf{vertex cover} \text{ of size } k\}.$$

A **vertex cover** of size k of $G = (V, E)$ is a subset $C \subseteq V$ of nodes, such that $|C| = k$ and for all $\{u, v\} \in E$ it holds that $u \in C$ or $v \in C$.

Show that VERTEXCOVER is **decidable** by giving an algorithm (abstract description or pseudo-code) that decides whether a graph has a vertex cover of size k or not *(6 points)*.

Explain why your algorithm accepts **exactly** the instances $\langle G, k\rangle$ which have a vertex cover of size k *(1 point)* and why it always halts *(1 points)*.

(b) Let $c \in \mathbb{N}$ be a fixed **constant** and let Σ be a **finite** alphabet.

Consider the **length-restricted** Halting problem defined over Σ.

$$H_c := \{\langle M, s\rangle \mid \text{Turing machine } M \text{ halts on input } s \text{ and } |\langle M, s\rangle| \leq c\},$$

H_c is similar to the usual Halting problem but restricted to strings $\langle M, s\rangle$ shorter than c.

State whether H_c is decidable or not *(2 points)*. Proof your claim *(5 points)*.

Task 6: Complexity Theory (12 Points)

Consider the following problems

$$\text{INDEPENDENTSET} := \{\langle G, k\rangle \mid \text{undirected, simple graph } G \text{ has an } \textbf{independent set} \text{ of size } k\}.$$

An **independent set** of size k of $G = (V, E)$ is a subset $I \subseteq V$ of nodes, such that $|I| = k$ and $\{u, v\} \notin E$ for all $u, v \in I, u \neq v$.

$$\text{CLIQUE} := \{\langle G, k\rangle \mid \text{undirected, simple graph } G \text{ has a } \textbf{clique} \text{ of size } k\}.$$

A **clique** of size k of $G = (V, E)$ is a subset $C \subseteq V$ of nodes, such that $|C| = k$ and $\{u, v\} \in E$ for all $u, v \in C, u \neq v$.

Use that CLIQUE is a **known** $\mathcal{NP}$-complete problem to **show** that INDEPENDENTSET is $\mathcal{NP}$-complete.

Remark: Document the steps of your proof carefully.

Task 7: Logic (15 Points)

(a) Consider the following **propositional logical** entailment

$$\{p \lor r \lor s, p \lor \neg r, \neg p \lor q, \neg q\} \models s \land \neg r$$

  (i) Use known equivalencies to convert the above entailment into the form $KB \models \bot$, where KB is in **Conjunctive Normal Form** (3 points).

  (ii) Use the **Resolution Calculus** to prove this logical entailment (5 points).

(b) Consider the following **first order logical** formula

$$\varphi := [\forall x \forall y\, f(s(x,y)) \doteq s(f(x), f(y))] \land [\exists x \exists y\, \neg(\, f(x) \doteq f(y)\,)]$$

where x, y are variable symbols and f, s are function symbols.

  (i) Give an interpretation which is **no model** of φ *(3 points)*.

  (ii) Give a **model** of φ *(4 points)*.

Remark: No proof required. Mark which interpretation is a model and which is not.