Exercise 1: Constructing DFAs

Construct DFAs that recognize the following languages. Drawing the state diagrams is sufficient. The alphabet is $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid |w| \geq 2$ and $w$ contains an odd number of ones\}.

(b) $L_2 = \{w \mid w$ contains an even number of zeros\}.

(c) $L_3 = \{in \ w \mid w$ every zero is immediately followed by a one\}.

(d) $L_4 = \{w \mid w$ ends with 01\}.

Sample Solution

1.

2.

3.
Exercise 2: Maxstring

Let

\[ \text{maxstring}(L) = \{ w : w \in L \text{ and for all words } z \in \Sigma^* : z \neq \epsilon \Rightarrow wz \notin L \} . \]

(a) What is \( \text{maxstring}(L_1L_2) \), where \( L_1 = \{ w \in \{a,b\}^* : w \text{ contains exactly one } a \} \) and \( L_2 = \{a\} \)?

(b) Explain how to prove that the regular languages are closed under \( \text{maxstring} \).

**Hint:** Let \( L \) be a regular language. You need to prove that \( \text{maxstring}(L) \) is regular as well.

**Sample Solution**

(a) \( L_1L_2 \). Let \( w \in L_1L_2 \) then it can be written as \( w = w_1w_2 \) with \( w_1 \in L_1 \), \( w_2 \in L_2 \), \( w_1 \) contains exactly one \( a \) and \( w_2 = a \). Assume there is some \( z \neq \epsilon \) with \( w_1w_2z = w_1az \in L_1L_2 \). As all words in \( L_1L_2 \) end with an \( a \) this implies \( z = a \) and \( w_1a \in L_1 \). However, as \( w_1 \) contains exactly one \( a \) the word \( w_1a \) contains two \( a \) and is not contained in \( L_1 \), a contradiction.

The inclusion \( \text{maxstring}(L_1L_2) \subseteq L_1L_2 \) is by definition.

(b) Take a DFA \( A \) for the language \( L \). Construct an DFA \( B \) for the language \( \text{maxstring}(L) \) by taking \( A \) with a modified set of accepting states: Let the set of accepting states of \( B \) be the accepting states of \( A \) that do not have a path to an accepting state.
Exercise 3: From NFA to DFA

Consider the following NFA.

(a) Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.

(b) Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

(c) Explain which language the automaton accepts.

Sample Solution

(a) The set of states is \( Q = \{q_0, q_1, q_2\} \); the alphabet \( \Sigma = \{a, b\} \); the starting state is \( q_0 \); the set of accept states is \( F = \{q_2\} \); the transition function is shown in the following table.

\[
\begin{array}{c|c|c|c}
   & q_0 & q_1 & q_2 \\
\hline
a & \{q_1\} & \emptyset & \emptyset \\
b & \{q_0, q_1\} & \{q_0, q_2\} & \{q_0\}
\end{array}
\]

(b) After performing the algorithm from the lecture we obtain the following DFA. All transitions which are not in the picture go to the garbage state \( \emptyset \).

(c) The recognized language contains words of length at least two. Furthermore any \( a \) is immediately followed by a \( b \). The number of \( b \)'s after the last \( a \) must not be two.