Exercise 1: \(\mathcal{O}\)-Notation

State whether the following claims are correct or not and prove it using the formal definition of the \(\mathcal{O}, \Omega, \Theta\)-notations.

(a) \(n(n-1) \in \mathcal{O}(n^2)\)
(b) \(n! \in \Omega(n^2)\)
(c) \(n \in \Theta(\log_2 3^n)\)
(d) \(\sqrt{n^3} \in \mathcal{O}(n \log n)\) \(\text{Hint: For all } \varepsilon > 0 \text{ there is an } n_0 \in \mathbb{N} \text{ such that for all } n \geq n_0 : \log_2 n \leq n^\varepsilon.\)

Exercise 2: Sort Functions by Asymptotic Growth

Use the definition of the \(\mathcal{O}\)-notation to give a sequence of the functions below, which is ordered by asymptotic growth (ascending). Between two consecutive functions \(g\) and \(f\) in your sequence, insert either \(\prec\) (in case \(g \in \mathcal{O}(f)\) and \(f \notin \mathcal{O}(g)\)) or \(\simeq\) (in case \(g \in \mathcal{O}(f)\) and \(f \in \mathcal{O}(g)\)).

\[
\begin{array}{cccc}
n^2 & \sqrt{n} & 2\sqrt{n} & \log(n^2) \\
2\sqrt{\log_2 n} & \log(n!) & \log(\sqrt{n}) & (\log n)^2 \\
\log n & 10^{100} n & n! & n \log n \\
2^n / n & n^n & \sqrt{\log n} & n \\
\end{array}
\]