University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
P. Bamberger

Algorithms Theory
Exercise Sheet 2

Exercise 1: Convolution *(11 Points)*

Compute the convolution of the vectors $a = (5, 8, -2, 3)$ and $b = (-9, 4, -1)$ using the algorithm for polynomial multiplication from the lecture. Document all computation steps for evaluation, point-wise multiplication and interpolation.

Exercise 2: Scheduling *(11 Points)*

Given n jobs of lengths $t_1 \dots, t_n$ with **one deadline** $d \geq 0$, we want to schedule these jobs such that the **average lateness** is minimized. That is, for each job i we want to find a start and finishing time $0 \leq s(i) \leq f(i)$ with $f(i) - s(i) = t_i$ such that the intervals $[s(i), f(i)]$ are pairwise non-overlapping and the average over all $L(i) = \max\{0, f(i) - d\}$ is minimal (overlapping of start- and endpoints is allowed).

Describe a greedy algorithm for this problem and prove that it computes an optimal solution.

Exercise 3: Matroids *(8 Points)*

We are given a directed weighted graph $G = (V, E)$, where $w : E \to \mathbb{R}^+$ defines weights of the edges. Consider also a function $b : V \to N$ that defines some indegree bound for each node. We would like to find a subset $E' \subseteq E$ of maximum total weight such that every node $u \in V$ has indegree at most $b(u)$ in the graph $G' = (V, E')$. Show that the set of feasible solutions form a matroid and thus, this problem can be solved by using the greedy algorithm for matroids.