Chapter 6
Graph Algorithms

Algorithm Theory
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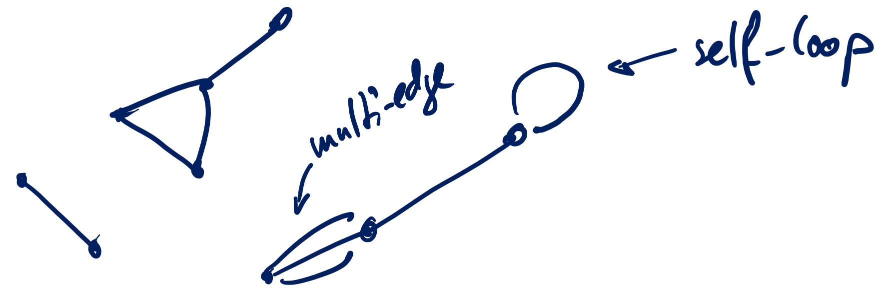
Graphs

Extremely important concept in computer science



Graph $G = (V, E)$

- V: node (or vertex) set
- $E \subseteq V^2$: edge set
  - Simple graph: no self-loops, no multiple edges
  - Undirected graph: we often think of edges as sets of size 2 (e.g., $\{u, v\}$)
  - Directed graph: edges are sometimes also called arcs
  - Weighted graph: (positive) weight on edges (or nodes)
- (simple) path: sequence $v_0, \dots, v_k$ of nodes such that
  $$(v_i, v_{i+1}) \in E \text{ for all } i \in \{0, \dots, k-1\}$$
- …

Many real-world problems can be formulated as optimization problems on graphs

Graph Optimization: Examples

Minimum spanning tree (MST):

- Compute min. weight spanning tree of a weighted undir. Graph

Shortest paths:

- Compute (length) of shortest paths (single source, all pairs, …)

Traveling salesperson (TSP):

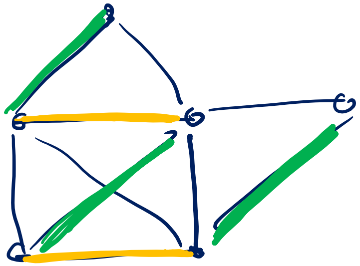
- Compute shortest TSP path/tour in weighted graph

Vertex coloring:

- Color the nodes such that neighbors get different colors
- Goal: minimize the number of colors

Maximum matching:

- Matching: set of pair-wise non-adjacent edges
- Goal: maximize the size of the matching



Network Flow

Flow Network:

- Directed graph $G = (V, E)$, $E \subseteq V^2$
- Each (directed) edge e has a capacity $c_e \geq 0$
  - Amount of flow (traffic) that the edge can carry
- A single source node $s \in V$ and a single sink node $t \in V$

Flow: (informally)

- Traffic from s to t such that each edge carries at most its capacity

Examples:

- Highway system: edges are highways, flow is the traffic
- Computer network: edges are network links that can carry packets, nodes are switches
- Fluid network: edges are pipes that carry liquid

Example: Flow Network