

Theoretical Computer Science - Bridging Course

Winter Term 2019/2020

Exercise Sheet 4

for getting feedback submit electronically by 12:15, Monday, November 18, 2019

Exercise 1: Context-Free Grammar (3+2 Points)

For each of the following languages, give a context-free grammar to accept the language.

(a) $L_1 = \{w\#w' \mid w^R \text{ is a substring of } w', \text{ and } w, w' \in \{0, 1\}^*\}$.¹

(b) $L_2 = \{0^i 1^j 2^k \mid i \neq j \text{ or } j \neq k\}$

Exercise 2: Chomsky Normal Form (5 Points)

Consider the following context-free grammar (CFG):

$$\begin{aligned} S &\rightarrow aSb \mid D \\ D &\rightarrow ccDcc \mid \varepsilon \end{aligned}$$

Convert this CFG into an equivalent one in Chomsky Normal Form. Give the grammar you obtained after each step of the conversion algorithm.

Exercise 3: Constructing Pushdown Automata (4 Points)

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$. Construct a PDA \mathcal{A} with $L(\mathcal{A}) = L$.

Exercise 4: Pumping Lemma for Context-Free Languages (3+3 Points)

Use the pumping lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not context free:

(a) $\{ww \mid w \in \{a, b\}^*\}$

(b) $\{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$

¹ w^R is achieved by reversing the order of the symbols in w .