

Theoretical Computer Science - Bridging Course

Winter Term 2019/2020

Exercise Sheet 5

for getting feedback submit electronically by 12:15, Monday, November 25, 2019

Exercise 1: The Shift Operation

(4+4 Points)

Consider a Turing machine \mathcal{M} that is given an arbitrary input string over alphabet $\Sigma = \{1, 2, \dots, n\}$ on its input tape. We would like \mathcal{M} to insert an empty cell, i.e., \sqcup , at the beginning of the tape without removing any symbol on the tape. As an example, the Turing machine is supposed to change the input tape of the form $\langle 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \dots \rangle$ to $\langle \sqcup, 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \dots \rangle$. Although this operation is not explicitly defined for a Turing machine, one can consider such an operation as shifting the whole string one cell to the right on the input tape.

- Give a formal definition of \mathcal{M} to perform the desired operation such that \mathcal{M} recognizes the language Σ^* .
- For $n = 2$, i.e., $\Sigma = \{1, 2\}$, draw the state diagram of your constructed Turing machine.

Exercise 2: Constructing Turing Machines I

(4+1+2+1 Points)

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over Σ .

- Give a state diagram of a Turing machine recognizing L .
- Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.
- Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.
- Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Constructing Turing Machines II

(4 Points)

Let $L = \{\langle w \rangle, \langle w + 1 \rangle \mid w \in \mathbb{N}\}$, e.g., the word $\langle 6 \rangle, \langle 7 \rangle = 110, 111$ is contained in L . Design a Turing machine which accepts L . You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

Remark: Here $\langle w \rangle$ is the binary encoding of the number w , e.g., the number 6 is going to be the string 110.