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Theoretical Computer Science - Bridging Course
Winter Term 2019/2020
Exercise Sheet 2

for getting feedback submit electronically by 12:15, Monday, November 04, 2019

Exercise 1: Drawing DFAs and NFAs *(8 Points)*

Consider the following three languages over the alphabet $\{0, 1\}$.

$L_1 = \{w \mid |w| \geq 2 \text{ and } w \text{ contains an even number of zeros}\}$.

$L_2 = \{w \mid w \text{ contains exactly two ones}\}$.

$L_3 = \{w \mid w \text{ has an odd number of zeros and ends with } 1\}$.

First draw a DFA for each of the languages L_1, L_2 and L_3. Then, for each of the following languages, provide an NFA that recognizes the given language.

(a) L_1^*

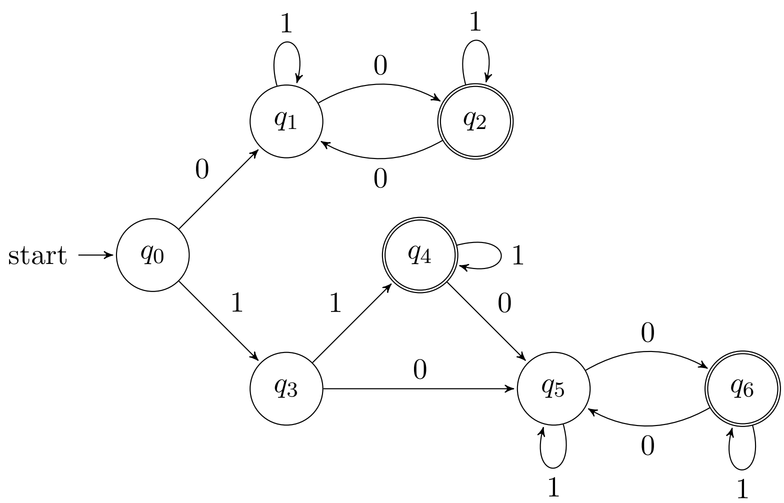
(b) $L_3 \circ L_2$

(c) $L_2 \cup L_3$

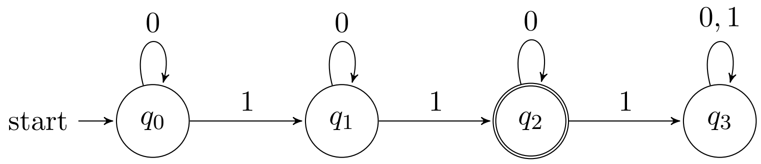
Sample Solution

Here are the DFAs for the three languages:

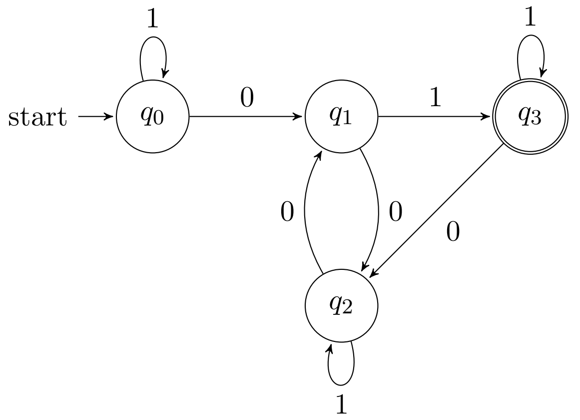
(a) L_1:



(b) L_2:



(c) L_3:



For constructing the NFAs regarding the given three languages in (a), (b), and (c), it is enough to reuse the drawn DFAs and insert proper epsilon transitions. Let N_1 and N_2 denote two DFAs. Then the following figures show how to utilize the DFAs to construct $L(N_1) \cup L(N_2)$, $L(N_1) \circ L(N_2)$, and $L(N_1)^*$ respectively. The figures are taken from the lecture slides.

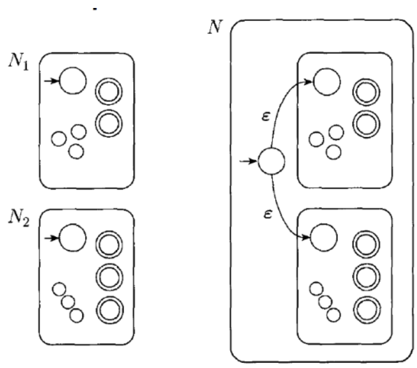


Figure 1: $L(N_1) \cup L(N_2)$

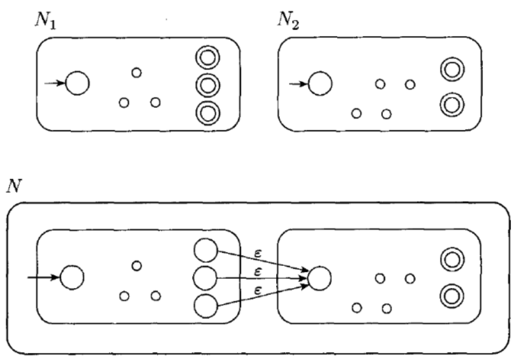


Figure 2: $L(N_1) \circ L(N_2)$

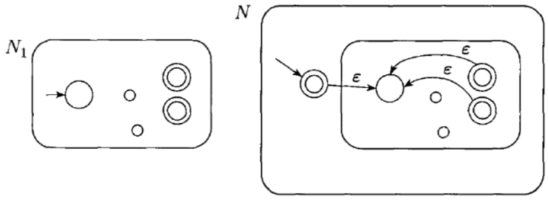


Figure 3: $L(N_1)^*$

Exercise 2: Regular Languages *(4 Points)*

Let L, L_1, L_2 be regular languages. Show that both $\overline{L} := \Sigma^* \setminus L$ and $L_1 \cap L_2$ are regular as well by constructing the corresponding DFAs.

Remark: *No need for drawing state diagrams. Show how a DFA for the language in question can be constructed presuming the existence of DFAs for L, L_1, L_2.*

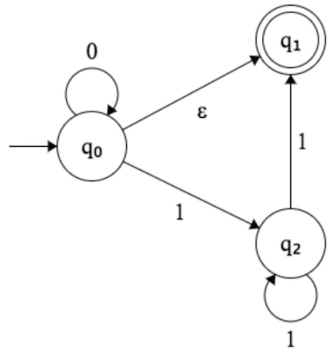
Sample Solution

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA recognizing L. We define the DFA $\overline{M} := (Q, \Sigma, \delta, q_0, \overline{F})$ by inverting the set of accepting states of M, i.e. $\overline{F} := Q \setminus F$. We show that $\overline{M}$ recognizes $\overline{L}$.
If $w \in \overline{L}$, then $w \notin L$ and so M halts in an non accepting state q when processing w. $\overline{M}$ will halt in the same state (because we only changed the set of accepting states), but here q is an accepting state. Analogously, if $w \notin \overline{L}$, then $w \in L$ and so M halts in an accepting state when processing w. $\overline{M}$ will again halt in the same state, but here q is a non accepting state. So we have that $\overline{M}$ halts in an accepting state when processing w if and only if $w \in \overline{L}$. Thus $\overline{M}$ recognizes the language $\overline{L}$ which is therefore regular.

For proving the regularity of $L_1 \cap L_2$, we construct the product automaton like done in the lecture (Theorem 1.25. p. 30) for $L_1 \cup L_2$, with the difference that we set $F := F_1 \times F_2$ as the set of accepting states, where F_1 and F_2 are the sets of accepting states of the DFAs for L_1 and L_2.

Exercise 3: NFA to DFA *(8 Points)*

Consider the following NFA.



(a) Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.

(b) Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

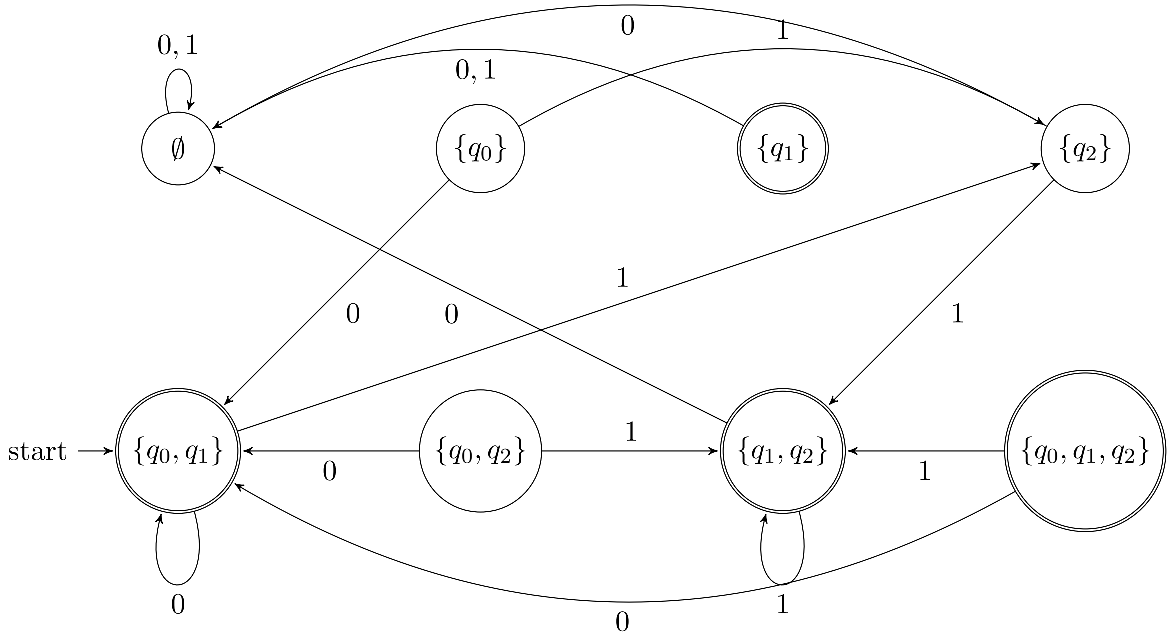
(c) Explain what language the automaton recognizes.

Sample Solution

(a) The set of states is $Q = \{q_0, q_1, q_2\}$; the alphabet $\Sigma = \{0, 1\}$; the initial state is q_0; the set of accept states is $F = \{q_1\}$; the transition function is shown in the following table.

	q_0	q_1	q_2
0	q_0	$\emptyset$	$\emptyset$
1	q_2	$\emptyset$	q_1, q_2
ϵ	q_1	$\emptyset$	$\emptyset$

(b)



If we leave out nodes with no path leading into it, he have